

19/MHS 01/402

TABUABO CHUKWUSOM TABSON (MBBS)

1)  $\int \frac{11-3x}{x^2+2x-3} dx$

Expressing  $\frac{11-3x}{x^2+2x-3}$  as a partial fraction

We have,

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$11-3x = A(x+3) + B(x-1)$$

Where  $x+3=0$   
 $x = -3$

Substitute  $x = -3$  into the eqn

$$11-3(-3) = A(-3+3) + B(-3-1)$$

$$11+9 = 0 - 4B$$

$$\frac{20}{-4} = \frac{-4B}{-4}$$

$$B = -5$$

where  $x-1=0$

$$x = 1$$

Substitute  $x = 1$  into the eqn

$$11-3(1) = A(1+3) + B(1-1)$$

$$11-3 = 4A + 0$$

$$8 = 4A$$

$$A = 2$$

$$A = \frac{2}{1}$$

So we have the partial fraction to be

$$\frac{2}{x-1} + \frac{-5}{x+3}$$

$$\int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx$$

since

$$\int \frac{A}{ax+b} dx = A \ln(ax+b) + C$$

We have

$$2 \ln(x-1) - 5 \ln(x+3)$$

We then have our final ans

to be  $2 \ln(x-1) - 5 \ln(x+3)$

$$\therefore \int \frac{11-3x}{x^2+2x-3} = 2 \ln(x-1) - 5 \ln(x+3)$$

2)  $\int \frac{4x+6}{x^2-2x-3}$

Resolve  $\frac{4x+6}{x^2-2x-3}$  into partial fraction

$$\frac{4x+6}{x^2-2x-3} = \frac{4x+6}{(x+1)(x-3)}$$

We have,

$$\frac{4x+6}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$4x+6 = A(x-3) + B(x+1)$$

When  $x = 3$

$$4(3)+6 = A(3-3) + B(3+1)$$

$$4x+6 = 0 + 4B$$

$$4(3)+6 = 4B$$

$$12+6 = 4B$$

$$\frac{18}{4} = \frac{4B}{4}$$

$$B = \frac{9}{2}$$

when  $x = -1$

$$4(-1) + 6 = A(-1-3) + B(-1+1)$$

$$\frac{-4 + 6}{-4} = \frac{-4A}{-4} + 0$$

$$A = \frac{1}{2}$$

We have,

$$\frac{-1}{2(x+1)} + \frac{9}{2(x-3)}$$

$$\int \frac{-1}{2(x+1)} + \int \frac{9}{2(x-3)}$$
$$-\frac{1}{2} \ln(2x+2) + \frac{9}{2} \ln(2x-6)$$

$$\frac{4x+6}{x^2-2x-3} = \frac{-1}{2} \ln(2x+2) + \frac{9}{2} \ln(2x-6)$$

$$\frac{4x+6}{x^2-2x-3} = \frac{-1}{2} \ln(2x+2) + \frac{9}{2} \ln(2x-6)$$

$$3) \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx$$

Resolve  $\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}$  into partial fractions

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

when  $x = -1$

$$2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3)$$

$$+ B(-1+1)(-1+3) + C(-1+1)(-1-2)$$

$$2 + 9 - 35 = A(-3)(2)$$

$$\frac{-24}{-6} = \frac{-6A}{-6}$$

$$A = 4$$

$$A = 4$$

when  $x-2=0$

$$x = 2$$

$$2(2)^2 - 9(2) - 35 = A(2-2)(2+3)$$

$$+ B(2+1)(2+3) + C(2+1)(2-2)$$

$$8 - 18 - 35$$

$$\frac{-45}{15} = \frac{15B}{15}$$

$$B = -3$$

When  $x+3=0$

$$x = -3$$

$$2(-3)^2 - 9(-3) - 35 = A(-3-2)$$

$$(-3+3) + B(-3+1)(-3+3)$$

$$+ C(-3+1)(-3-2)$$

$$18 + 27 - 35 = C(-2)(-5)$$

$$\frac{10}{-10} = \frac{10C}{-10}$$

$$C = 1$$

We have,

$$\frac{4}{x+1} - \frac{3}{x-2} + \frac{1}{x+3}$$

$$\int \frac{4}{x+1} dx - \int \frac{3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$= 4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3|$$