

SHERIFF FATIMA

19/MHS01/398

$$1 \int \frac{11-3x}{x^2+2x-3} dx = \int \frac{11-3x}{(x-1)(x+3)} dx$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$11-3x = A(x+3) + B(x-1)$$

$$p(1) \Rightarrow 8 = 4A \quad \therefore A = 2$$

$$p(-3) \Rightarrow 20 = -4B \quad \therefore B = -5$$

$$\int \frac{11-3x}{x^2+2x-3} dx = \int \frac{2}{x-1} dx - \int \frac{5}{x+3} dx = 2 \int \frac{dx}{x-1} - 5 \int \frac{dx}{x+3} = 2 \ln|x-1| - 5 \ln|x+3| + C$$

$$\int \frac{11-3x}{x^2+2x-3} dx = 2 \ln|x-1| - 5 \ln|x+3| + C$$

$$2 \int \frac{4x-16}{x^2-2x} dx = \int \frac{4x-16}{x(x-2)}$$

$$\frac{4x-16}{x^2-2x} = \frac{A}{x} + \frac{B}{x-2}$$

$$4x-16 = A(x-2) + Bx$$

$$p(0) \Rightarrow -16 = -2A \quad \therefore A = 8$$

$$p(2) \Rightarrow -8 = 2B \quad \therefore B = -4$$

$$\int \frac{4x-16}{x^2-2x} dx = \int \frac{8}{x} dx - \int \frac{4}{x-2} dx = 8 \int \frac{1}{x} dx - 4 \int \frac{1}{x-2} dx$$
$$= 8 \ln|x| + 4 \ln|x-2| + C$$

$$3 \int \frac{2x^2-9x-35}{(x-1)(x-2)(x+3)} dx$$

$$\frac{2x^2-9x-35}{(x-1)(x-2)(x+3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$2x^2-9x-35 = A(x-2)(x-3) + B(x-1)(x+3) + C(x-1)(x-2)$$

$$2x^2-9x-35 = A(x^2-5x+6) + B(x^2+2x-3) + C(x^2-3x+2)$$

$$2x^2-9x-35 = Ax^2-5Ax+6A+Bx^2+2Bx-3B-Cx^2-3Cx+2C$$

$$2x^2-9x-35 = Ax^2+Bx^2+Cx^2+5Ax+2Bx-3Cx+6A-3B+2C$$

$$2x^2-9x-35 = (A+B+C)x^2 + (5A+2B-3C)x + (6A-3B+2C)$$

$$A+B+C = 2 \quad \text{--- (1)}$$

$$5A+2B-3C = -9 \quad \text{--- (2)}$$

$$6A-3B+2C = -35 \quad \text{--- (3)}$$

$$A = 2 - B - C \quad \text{--- (4)}$$

$$5(2 - B - C) + 2B - 3C = -9$$

$$10 - 5B - 5C + 2B - 3C = -9$$

$$-3B - 3C = -19 \quad \text{--- (5)}$$

$$6(2 - B - C) - 3B + 2C = -35$$

$$12 - 6B - 6C - 3B + 2C = -35$$

$$-9B - 4C = -47 \quad \text{--- (6) } \times 2$$

$$-18B - 8C = -94 \quad \text{--- (7) } \times 1$$

$$-18B - 8C = -94 \quad \text{--- (8)}$$

$$\text{(8) - (7)} \rightarrow -3B - 8C = -19 \quad \text{--- (9)}$$

$$-15B = -75 \quad \therefore \boxed{B = 5}$$

$$\text{From (5): } -15 - 8C = -19$$

$$\boxed{C = 1/2}$$

$$\text{From (4): } A = 2 - 5 - 1/2 = -7/2 \quad \therefore \boxed{A = -7/2}$$

$$\int \frac{2x^2 - 9x - 35}{(x-1)(x-2)(x+3)} dx = \int \frac{-7/2}{x-1} dx + \int \frac{5}{x-2} dx + \int \frac{1/2}{x-3} dx$$

$$= \frac{-7}{2} \int \frac{dx}{x-1} + 5 \int \frac{dx}{x-2} + \frac{1}{2} \int \frac{dx}{x-3}$$

$$= \frac{-7}{2} \ln|x-1| + 5 \ln|x-2| + \frac{1}{2} \ln|x+3| + C$$