

Name: Jekami Mobolaji Deborah

Matric no: 19/MHS01/218

Department: Medicine

Assignment

1) $\int \frac{11-3x}{x^2+2x-3}$

Solution

$$\int \frac{11-3x}{(x+1)(x+3)} dx$$

$$\frac{11-3x}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{11-3x}{x^2+2x-3} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

Divide all through by $(x-1)(x+3)$

$$11-3x = A(x+3) + B(x-1)$$

When $x = -3$

$$11 - 3(-3) = A(-3+3) + B(-3-1)$$

$$11 + 9 = B(-3-1)$$

$$11 + 9 = -4B$$

$$B = \frac{-20}{4}$$

$$B = -5$$

When $x = 1$

$$11 - 3(1) = A(1+3) + B(1-1)$$

$$11 - 3 = A(1+3)$$

$$8 = 4A$$

$$A = 2$$

$$\int \frac{11-3x}{(x-1)(x+3)} dx = \frac{2}{x-1} + \frac{-5}{x+3}$$

$$\int \frac{11-3x}{x^2+2x-3} dx = 2 \ln|x-1| - 5 \ln|x+3| + C$$

$$2) \int \frac{4x-16}{x^2-2x-3} dx =$$

Solution.

$$\int \frac{4x-16}{(x-3)(x+1)}$$

$$4x-16 = A + B$$

$$x^2-2x-3 \quad (x-3)(x+1)$$

$$A(x+1) + B(x-3) = 4x-16$$

$$(x-3)(x+1) \quad (x-3)(x+1)$$

Multiply all through by $(x-3)(x+1)$

$$4x-16 = A(x+1) + B(x-3)$$

$$4(-) \text{ when } x = -1$$

$$4(-1) - 16 = A(-1+1) + B(-1-3)$$

$$-20 = -4B$$

$$B = 5$$

$$\text{When } x = 3$$

$$4(3) - 16 = A(3+1) + B(3-3)$$

$$12 - 16 = A(3+1)$$

$$-4 = 4A$$

$$A = \frac{-4}{4}$$

$$A = -1$$

$$\therefore \int \frac{4x-16}{x^2-2x-3} = \frac{-1}{x-3} + \frac{5}{x+1} dx$$

$$\text{Let } u = x-3$$

$$\frac{du}{dx} = 1$$

$$dx = du$$

$$\text{Let } u = x+1$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$\therefore \frac{-1}{u} du + \frac{5}{u} du$$

$$\int \frac{4x-16}{x^2-2x-3} = -\ln(x-3) + 5\ln(x+1) + c$$

$$\int \frac{4x-16}{x^2-2x-3} dx = 5\ln(x+1) - \ln(x-3) + c$$

$$2) \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{A(x-2)(x+3) + B[(x+1)(x+3)] + C[(x+1)(x-2)]}{(x+1)(x-2)(x+3)}$$

$$\frac{A[x^2 + 3x - 2x - 6] + B[x^2 + 3x + x + 3] + C[x^2 - 2x + x - 2]}{(x+1)(x-2)(x+3)}$$

$$\frac{Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - 2Cx - 2C}{(x+1)(x-2)(x+3)}$$

$$\frac{x^2[A+B+C] + x[A+4B-C] + [-6A+3B-2C]}{(x+1)(x-2)(x+3)}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{x^2[A+B+C] + x[A+4B-C] + [-6A+3B-2C]}{(x+1)(x-2)(x+3)}$$

Multiply through by $(x+1)(x-2)(x+3)$

$$2x^2 - 9x - 35 = x^2(A+B+C) + x[A+4B-C] + [-6A+3B-2C]$$

$$A + B + C = 2 \quad \text{--- (1)}$$

$$A + 4B - C = -9 \quad \text{--- (2)}$$

$$-6A + 3B - 2C = 35 \quad \text{--- (3)}$$

$$A = 2 - B - C \quad \text{--- (4)}$$

Put eq. 4 into 2 and 3

$$2 - B - C + 4B - C = -9$$

$$2 - B + 4B - C - C = -9$$

$$2 + 3B - 2C = -9$$

$$3B - 2C = -9 - 2$$

$$3B - 2C = -11 \quad \text{--- (5)}$$

$$-6A + 3B - 2C = -35$$

$$-6(2 - B - C) + 3B - 2C = -35$$

$$-12 + 6B + 6C + 3B - 2C = -35$$

$$6B + 3B + 6C - 2C = -35 + 12$$

$$9B + 3C = 23 \quad \text{--- (6)}$$

Solve 5 & 6

$$3B - 3C = -11 \quad \text{--- (5)}$$

$$9B - 4C = -23 \quad \text{--- (6)}$$

Multiply eq. 5 by 4 and Eq. 6 by 2

$$12B - 8C = -44$$

$$18B + 8C = -46$$

$$30B \quad \quad -90$$

$$B = -90/30$$

$$B = -3 \quad \text{--- (7)}$$

Put Eq. 7 into 5

$$3B - 2C = -11$$

$$3(-3) - 2C = -11$$

$$-9 - 2C = -11$$

$$-2C = 2$$

$$C = 1 \quad \text{--- (8)}$$

Put eq 7 & 8 in eq. 1

$$A = 2 - B - C$$

$$= 2 - (-3) - 1$$

$$A = 4$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} + \frac{-3}{x-2} + \frac{1}{x+3}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} dx + \int \frac{-3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = 4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3| + C$$