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19/MHS01/393
Medicine and surgery
MAT 104

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Assignment

(1) $\int \frac{11-3x}{x^2+2x-3}\,dx$

$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$

$\Rightarrow \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$

$\therefore A(x+3) + B(x-1) = 11-3x$

At $x = -3$, we have

$B(-4) = 11+9$

$B = -5$

At $x = 1$, we have

$A(1+3) = 11-3(1)$

$4A = 8$

$A = 2$

We can now write

$\int \frac{2}{x-1}\,dx + \int \frac{-5}{x+3}\,dx = \int \frac{11-3x}{x^2+2x-3}\,dx$

$\Rightarrow \int \frac{2\,dx}{x-1} + \int \frac{-5\,dx}{x+3} = \int \frac{11-3x}{x^2+2x-3}\,dx$

Let $u = x-1$ $\quad u = x+3$

$du = dx$ $\quad du = dx$

$\Rightarrow \int \frac{2\,du}{u}$ $\quad -5\int \frac{du}{u}$

$= 2\ln u$ $\quad -5\ln u$

$\therefore \int \frac{11-3x}{x^2+2x-3}\,dx = 2\ln(x-1) - 5\ln(x+3)$

(2) $\int \frac{4x-16}{x^2-2x-3}$

$= \frac{4x-16}{x^2-2x-3} = \frac{4x-16}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)}$

$= \frac{A(x+1)+B(x-3)}{(x-3)(x+1)}$

$= A(x+1)+B(x-3) = 4x-16$

$\therefore Ax+A+Bx-3B = 4x-16$

$(A+B)x+(A-3B) = 4x-16$

$A+B = 4$

$-\ A-3B = -16$

$4B = -12$

$B = -3$

$A+B = 4 \quad (\times 3)$

$A-3B = -16$

$+$

$3A+3B = 12$

$A-3B = -16$

$4A = -4$

$\therefore A = -1$

We can now write

$\int \frac{-1}{x-3}dx + \int \frac{-3}{x+1}dx = \int \frac{4x-16}{x^2-2x-3}dx$

$\Rightarrow \int \frac{-1\,dx}{x-3} + \int \frac{-3\,dx}{x+1} = \int \frac{4x-16}{x^2-2x-3}dx$

Let $u = x-3$ $\quad u = x+3$

$du = dx$ $\quad du = dx$

$\Rightarrow \int \frac{-1\,du}{u} \quad \int \frac{-3\,du}{u}$

$\therefore \int \frac{4x-16}{x^2-2x-3} = -1\ln(x-3) - 3\ln(x+1)$

(5) $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx$

$= \frac{2x^2-9x-35}{(x+1)(x-2)(x+3} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$

$= \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$

$= A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2) = 2x^2-9x-35$

Let $x=-1$, we have

$A(-3)(2) = 2(-1)^2 - 9(-1) - 35$

$-6A = -24$

$\therefore A = \frac{-24}{-6}$

$A = 4$

Let $x=2$, Then we have,

$B(3)(5) = 2(2)^2 - 9(2) - 35$

$15B = -45$

$B = \frac{-45}{15}$

$\therefore B = -3$

Let $x=-3$, then we have

$C(-2)(-5) = 2(-3)^2 - 9(-3) - 35$

$10C = 10$

$\therefore C = 1$

We can now write

$\int \frac{4}{x+1}\,dx + \int \frac{-3}{x-2}\,dx + \int \frac{1}{x+3}\,dx = \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx$

$\Rightarrow \int \frac{4\,dx}{x+1} + \int \frac{-3\,dx}{x-2} + \int \frac{dx}{x+3} = \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3}\,dx$

Let $u = x+1$	$u = x-2$	$u = x+3$
$du = dx$	$du = dx$	$du = dx$
$= 7\int \frac{4\,du}{u}$	$\int \frac{-3\,du}{u}$	$\int \frac{du}{u}$
$= 4\ln u$	$= -3\ln u$	$\ln u$

$\therefore \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3)$