Reg No: 19/MHS01/012
Dept: MBBS
Age: 17th,
Name: Adeba o Jeremiah

MAT Assignment

1 $\int \frac{11-3x}{(x^2+2x-3)} dx$

Solution

x^2+2x-3
$x^2+3x-x-3$
$x(x+3)-1(x+3)$
$(x-1)(x+3)$

$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)}$

$= \frac{A}{(x-1)} + \frac{B}{(x+3)} \Rightarrow \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$

$A(x+3)+B(x-1) = 11-3x$
$Ax+3A+Bx-B = 11-3x$
$(A+B)x+(3A-B) = 11-3x$

(Comparing)
$A+B = -3 \quad \cdots (1)$
$3A-B = 11 \quad \cdots (2)$

$4A = 8$
$\frac{4A}{4} = \frac{8}{4} \Rightarrow A = 2$

(Put in equation (1))
$2+B = -3$
$B = -3-2 \Rightarrow B = -5$

We can now write

$\int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx = \int \frac{11-3x}{x^2+2x-3}$

Let $u = x-1$ $\quad u = x+3$
$du = dx$ $\quad du = dx$

We can now write

$$\int \frac{2}{x-1}\,dx + \int \frac{-5}{x+3}\,dx = \int \frac{11-3x}{x^2+2x-3}$$

let $u = x-1$ $\quad u = x+3$

$du = dx$ $\quad du = dx$

$\Rightarrow \int \frac{2\,du}{u}$ $\quad \Rightarrow \int \frac{-5\,du}{u}$

$= 2\ln u$ $\quad \Rightarrow -5\ln u$

$\Rightarrow 2\ln(x-1) - 5\ln(x+3) + C$

2 $\int \frac{4x-16}{(x^2-2x-3)} dx$

Solution

$\int \frac{4x-16}{(x^2-2x-3)} dx \Rightarrow \int \frac{4x-16}{(x+1)(x-3)}$

x^2-2x-3
$x^2-3x+x-3$
$x(x-3)+1(x-3)$
$(x+1)(x-3)$

$= \frac{A}{x+1} + \frac{B}{x-3} \Rightarrow \frac{A(x-3)+B(x+1)}{(x+1)(x-3)}$

$A(x-3)+B(x+1) = 4x-16$

$Ax-3A+Bx+B = 4x-16$

$(B+A)x + (B-3A) = 4x-16$

(Comparing sides) $B+A = 4 \quad \ldots (1)$

$B-3A = -16 \quad \ldots (2)$

$4A = 20 \Rightarrow A = 5$

(Put in equation (1))

$B+5 = 4$

$B = 4-5 \Rightarrow B = -1$

We can now write

$\int \frac{5}{x+1} dx + \int \frac{-1}{x-3} dx = \int \frac{4x-16}{x^2-2x-3} dx$

let $u = x+1 \qquad u = x-3$

$du = dx \qquad du = dx$

$\Rightarrow \int \frac{5du}{u} \qquad \Rightarrow \int \frac{-du}{u}$

$\Rightarrow 5 \ln u \qquad \Rightarrow -\ln u$

$\Rightarrow 5\ln(x+1) - \ln(x-3) + c$

3 $\int \frac{(2x^2-9x-35)}{(x+1)(x-2)(x+3)}\,dx$

Solution

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{x+3}$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3)+B(x+1)(x+3)+C(x+1)(x-2)}{(x+1)(x+3)(x-2)}$$

$2x^2-9x-35 = A(x^2+x-6)+B(x^2+4x+3)+C(x^2-x-2)$

$2x^2-9x-35 = Ax^2+Ax-6A+Bx^2+4Bx+3B+Cx^2-Cx-2C$

$2x^2-9x-35 = Ax^2+Bx^2+Cx^2+Ax+4Bx-Cx-6A+3B-2C$

$2x^2-9x-35 = (A+B+C)x^2+(A+4B-C)x+(-6A+3B-2C)$

(Comparing) $A+B+C=2 \quad \cdots ①$

$A+4B-C=-9 \quad \cdots ②$

$-6A+3B-2C=-35 \quad \cdots ③$

$A+B+C=2$

$A+4B-C=-9 \quad \cdots ④$

$-3B+2C=11 \quad \therefore 3B-2C=-11$ (R in equation ③)

$\therefore -6A-(-11)=-35$

$-6A=-35+11=+24 \quad \therefore A=4$

$\therefore$ $4+B+C=2 \Rightarrow B+C=2-4 \Rightarrow B+C=-2$

$4+4B-C=-9 \Rightarrow 4B-C=-9-4 \Rightarrow 4B-C=-13$

$+\quad B+C=-2 \quad \ldots (5)$

$\quad\; 4B-C=-13 \quad \ldots (6)$

$\frac{5B}{5}=\frac{11}{5}$; $\therefore B=\frac{11}{5}$ (put in equation (5))

Finally, $\frac{11}{5}+C=-2$

$C=-2-\frac{11}{5}$

$C=\frac{-10-11}{5} \Rightarrow -\frac{21}{5}$

$\therefore A=4,\ B=\frac{11}{5},\ C=-\frac{21}{5}$

$$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx = \int \frac{4\,dx}{x+1} + \int \frac{\frac{11}{5}\,dx}{x-2} + \int \frac{-\frac{21}{5}\,dx}{x+3}$$

let $u=x+1$ $\quad u=x-2$ $\quad u=x+3$

let $u = x+1$ $\quad u = x-2$ $\quad u = x+3$

$du = dx$ $\quad du = dx$ $\quad du = dx$

$\Rightarrow 4\int \frac{du}{u} \quad \frac{11}{5}\int \frac{du}{u} \quad -\frac{21}{5}\int \frac{du}{u}$

$\Rightarrow 4\ln u \quad +\frac{11}{5}\ln u \quad -\frac{21}{5}\ln u$

$\Rightarrow 4\ln(x+1) + \frac{11}{5}\ln(x-2) - \frac{21}{5}\ln(x+3)$