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MATRIC NO: 19 [MMS01/131

$$1 \int \frac{11-3x}{x^2+2x-3}$$

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

Finding the L.C.M of R.H.S

$$\frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

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$$\frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

Multiply through by $(x-1)(x+3)$

$$11-3x = A(x+3) + B(x-1)$$

When $x = -3$

$$B(-3-1) = 11-3(-3)$$

$$B(-4) = 11+9$$

$$B(-4) = 20$$

$$B = -5$$

When $x = 1$

$$A(1+3) = 11-3(1)$$

$$A(4) = 8$$

$$A = 2$$

We can now write

$$\int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx = \int \frac{11-3x}{x^2+2x-3} dx$$

$$\text{let } u = x - 1$$

$$du = dx$$

$$dx = du$$

$$2 \int \frac{du}{u}$$

$$\text{let } u = x + 3$$

$$du = dx$$

$$dx = du$$

$$-5 \int \frac{du}{u}$$

$$= \int \frac{11-3x}{x^2+2x-3} dx$$

$$\therefore \int \frac{11-3x}{x^2+2x-3} dx = 2 \ln|x-1| - 5 \ln|x+3|$$

$$2 \int \frac{4x+6}{x^2-2x-3}$$

$$\frac{4x+6}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3}$$

Find the lcm of the R.H.S

$$\frac{A}{x+1} + \frac{B}{x-3}$$

$$\frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$$

$$\frac{4x+6}{(x+1)(x-3)} = \frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$$

Multiply through by $(x+1)(x-3)$

$$4x+6 = A(x-3) + B(x+1)$$

$$\text{when } x = 3$$

$$B(3+1) = 4(3)+6$$

$$B(4) = 12+6$$

$$B(4) = 18$$

$$B = \frac{18}{4} = 4.5$$

$$\text{When } x = -1$$

$$A(-1-3) = 4(-1)+6$$

$$A(-4) = -4+6$$

$$A(-4) = 2$$

$$A = -0.5$$

We can now write

$$\int \frac{-5}{x+1} dx + \int \frac{4}{x-3} dx = \int \frac{4x+6}{x^2-2x-3} dx$$

$$\text{let } u = x+1$$

$$du = dx$$

$$dx = du$$

$$= \int \frac{-5}{u} du$$

$$\text{let } u = x-3$$

$$du = dx$$

$$dx = du$$

$$= \int \frac{4}{u} du$$

$$= \int \frac{4x+6}{x^2-2x-3} dx$$

$$\therefore \int \frac{4x+6}{x^2-2x-3} dx = \frac{5}{5} \ln u - \ln u$$

$$= \frac{1}{5} \ln(5u) - \ln u$$

$$= \frac{1}{5} \ln(5(x+1)) - \ln(x-3)$$

$$3 \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

Find the LCM of the R.H.S

$$\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

Multiply through by $(x+1)(x-2)(x+3)$

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

$$2x^2 - 9x - 35 = A(x^2+x-6) + B(x^2+4x+3) + C(x^2-x-2)$$

$$2x^2 - 9x - 35 = Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$2x^2 - 9x - 35 = Ax^2 + Bx^2 + Cx^2 + Ax + 4Bx - Cx - 6A + 3B - 2C$$

$$2x^2 - 9x - 35 = (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C)$$

$$A + B + C = 2 \quad \dots \text{(i)}$$

$$A + 4B - C = -9 \quad \dots \text{(ii)}$$

$$-6A + 3B - 2C = -35 \quad \dots \text{(iii)}$$

$$A = 2 - B - C \quad \text{From eqn (i) and put in (ii)}$$

$$2 - B - C + 4B - C = -9$$

$$3B - 2C = -11$$

$$3B = -11 + 2C \quad \text{Put in eqn (iii)}$$

$$-6A - 11 + 2C - 2C = -35$$

$$-6A = -24$$

$$A = 4$$

$$\text{Recall } 3B = -11 + 2C$$

$$B = \frac{-11 + 2C}{3} \quad \text{Put in eqn (i)}$$

$$4 + \left[\frac{-11 + 2C}{3} \right] + C = 2$$

$$4 - \left[\frac{11 + 2C}{3} \right] + C = 2$$

Multiply through by 3

$$12 - 11 + 2C + 3C = 6$$

$$1 + 5C = 6$$

$$5C = 5$$

$$C = 1 \quad \text{Put into any equation}$$

$$4 + B + 1 = 2$$

$$B + 5 = 2$$

$$B = -3$$

$$\int \frac{4 dx}{x+1} + \int \frac{-3 dx}{x-2} + \int \frac{1 dx}{x+3} = \int \frac{2x^2 - 9x - 35 dx}{(x+1)(x-2)(x+3)}$$

$$\text{let } u = x+1$$

$$du = dx$$

$$dx = du$$

$$4 \ln u$$

$$\text{let } u = x-2$$

$$du = dx$$

$$dx = du$$

$$-3 \ln u$$

$$\text{let } u = x+3$$

$$du = dx$$

$$dx = du$$

$$\ln u$$

$$\therefore \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)$$