AMUSAN BLOSSOM OLUWATIMILEHIN
19/MHS01/093
MEDICINE & SURGERY
MAT 104

1. $\int \frac{11-3x}{x^2+2x-3}\,dx$.

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)}.$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}.$$

Find the L.C.M. of the right hand side

$$\frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

Multiply through by $(x-1)(x+3)$

$$11-3x = A(x+3)+B(x-1)$$

When $x=-3$.

$$B(-3-1) = 11-3(-3).$$

$$B(-4) = 11+9.$$

$B(-4) = 20$

$B = -6$

when $x = 1$.

$A(1+3) = 11 - 3(1)$

$A(4) = 8$

$A = 2$.

To now write,

$$\int \frac{2}{x-1}\,dx + \int \frac{-5}{x+3}\,dx = \int \frac{11-3x}{x^2+2x-3}\,dx$$

let $u = x-1$

$du = dx$.

$dx = du$.

$2\int \frac{du}{u}$

let $u = x+3$

$du = dx$

$dx = du$

$-5\int \frac{du}{u}$

$$\therefore \int \frac{11-3x}{x^2+2x-3}\,dx = 2\ln(x-1) - 5\ln(x+3)$$

2. $\int \frac{4x+16}{x^2-2x-3}$

$\frac{4x-16}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3}$

find the L.C.M of the R.H.S.

$\frac{A}{x+1} + \frac{B}{x-3}$

$\frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$.

$\frac{4x-16}{(x+1)(x-3)} = \frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$

Multiply through by $(x+1)(x-3)$

$4x-16 = A(x-3) + B(x+1)$.

when $x = 3$.

$B(3+1) = 4(3) - 16$.

$B(4) = -4$

$B = -1$.

when $x = -1$

$A(-1-3) = 4(-1) - 16$

$A(-4) = -20$

$A = 5$.

We can now write

$$\int \frac{5}{x+1}\,dx + \int \frac{-1}{x-3}\,dx = \int \frac{4x-16}{x^2-2x-3}\,dx$$

Let $u = x+1$.

$du = dx$

$dx = du$

$$5\int \frac{du}{u}$$

Let $u = x-3$

$du = dx$

$dx = du$.

$$-1\int \frac{du}{du} = \int \frac{4x-16}{x^2-2x-3}\,dx$$

$$\therefore \int \frac{4x-16}{x^2-2x-3} = 5\ln u - \ln u$$

$$= 5\ln(x+1) - \ln(x-3)$$

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3 $$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx$$

$$\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

Find the LCM of the RHS

$$\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

Multiply through by $(x+1)(x-2)(x+3)$

$2x^2-9x-35 = A(x+3)(x-2)+B(x+1)(x+3)+C(x+1)(x-2)$

$2x^2-9x-35 = A(x^2+x-6)+B(x^2+4x+3)+C(x^2-x-2)$

~~$2x^2-9x-35 = Ax^2$~~

$2x^2-9x-35 = Ax^2+Ax-6A+Bx^2+4Bx+3B+Cx^2-Cx-2C$

$2x^2-9x-35 = Ax^2+Bx^2+Cx^2+Ax+4Bx-Cx-6A+3B-2C.$

$2x^2-9x-35 = (A+B+C)x^2+(A+4B-C)x+(C-6A+3B-2C)$

$A+B+C = 2 \quad \text{---(i)}$

$A+4B-C = -9 \quad \text{---(ii)}$

$-6A+3B-2C = -35 \quad \text{---(iii)}$

$A = 2-B-C$ from eqn (i)

putting in eqn (i) in eqn (ii)

$2-B-C+4B-C = -9$

$3B-2C = -11$

$3B = -11+2C$ (put in eqn (iii)).

$-6A-11+2C-2C = -35$

$-6A-11 = -35.$

$-6A = -35+11$

$-6A = -24$

$A = 4$

Recall $3B = -11+2C.$

$B = \frac{-11+2C}{3}$

put in eqn (i)

$A + B + C = 2$

Since $A = 4$

$4 + \left[\frac{-11 + 2C}{3}\right] + C = 2$

$4 \frac{-11 + 2C}{3} + C = 2.$

Multiply through by 3

$12 - 11 + 2C + 3C = 6.$

$12 - 11 + 5C = 6$

$1 + 5C = 6$

$5C = 6 - 1$

$5C = 5$

$C = 1$

Now put $(C = 1)$ in any equation

$4 + B + 1 = 2$ from eqn (i)

$5 + B = 2.$

$B = 2 - 5$

$B = -3.$

$$\int \frac{4\,dx}{x+1} + \int \frac{-3\,dx}{x-2} + \int \frac{1}{x+3}\,dx = \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx$$

let $u = x+1$	let $a = x-2$	let $u = x+3$
$du = dx$	$du = dx$	$du = dx$
$dx = du$	$dx = du$	$dx = du$
$4\ln u$	$-3\ln u$	$\ln u$

$$\therefore \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx = 4\ln(x+1) - 3(x-2) + \ln(x+3)$$