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19/mhsoil152

mat104

mbbs

$$\textcircled{1} \int \frac{11-3x}{x^2+2x-3} dx$$

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

Lcm of the right hand side

$$\frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

Multiply through by  $(x-1)(x+3)$

$$11-3x = A(x+3) + B(x-1)$$

when  $x = -3$

$$B(-3-1) = 11-3(-3)$$

$$B(-4) = 11+9$$

$$B(-4) = 20$$

$$B = -5$$

when  $x = 1$

$$A(1+3) = 11-3(1)$$

$$A(4) = 8$$

$$A = 2$$

So therefore

$$\int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx = \int \frac{11-3x}{x^2+2x-3} dx$$

$$\text{let } u = x-1$$

$$du = dx$$

$$dx = du$$

$$2 \int \frac{du}{u}$$

$$\text{let } u = x+3$$

$$du = dx$$

$$dx = du$$

$$-5 \int \frac{du}{u}$$

$$\therefore \int \frac{11-3x}{x^2+2x-3} dx = 2 \ln|x-1| - 5 \ln|x+3|$$

$$2) \int \frac{4x-16}{x^2-2x-3}$$

$$\frac{4x-16}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3}$$

Lcm of Right hand Side

$$\frac{A}{x+1} + \frac{B}{x-3}$$

$$\frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$$

$$\frac{4x-16}{(x+1)(x-3)} = \frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$$

Multiply through with  $(x+1)(x-3)$

$$4x-16 = A(x-3) + B(x+1)$$

when  $x = 3$

$$B(3+1) = 4(3)-16$$

$$B(4) = -4$$

$$B = -1$$

when  $x = -1$

$$A(-1-3) = 4(-1)-16$$

$$A(-4) = -20$$

$$A = 5$$

So therefore

$$\int \frac{5}{x+1} dx + \int \frac{-1}{x-3} dx = \int \frac{4x-16}{x^2-2x-3} dx$$

$$\text{let } u = x+1 \quad \text{let } v = x-3$$

$$du = dx \quad dv = dx$$

$$dx = du \quad dx = dv$$

$$5 \int \frac{du}{u} - \int \frac{dv}{v} = \int \frac{4x-16}{x^2-2x-3} dx$$

$$= \int \frac{4x-16}{x^2-2x-3} = 5 \ln|x+1| - \ln|x-3|$$

$$3) \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx$$
$$\frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

LCM of Right hand side

$$\frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$\frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

Multiply through by  $(x+1)(x-2)(x+3)$

$$2x^2 - 9x - 35 = A(x+3)(x-2) + B(x+1)(x+3) + C(x+1)(x-2)$$

$$2x^2 - 9x - 35 = A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2)$$

$$2x^2 - 9x - 35 = Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$2x^2 - 9x - 35 = (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C)$$

$$2x^2 - 9x - 35 = (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C)$$

$$A + B + C = 2 \quad \dots \text{ (i)}$$

$$A + 4B - C = -9 \quad \dots \text{ (ii)}$$

$$-6A + 3B - 2C = -35 \quad \dots \text{ (iii)}$$

$$A = 2 - B - C \text{ from eqn (6)}$$

Putting eqn (6) in eqn (2)

$$2 - B - C + 4B - C = -9$$

$$3B - 2C = -11$$

$$3B = -11 + 2C \quad (\text{Put it in eqn (3)})$$

$$-6A - 11 + 2C - 2C = -35$$

$$-6A - 11 = -35$$

$$-6A = -35 + 11$$

$$-6A = -24$$

$$A = 4$$

Remember  $3B = -11 + 2C$

$$B = \frac{-11 + 2C}{3}$$

Put it in eqn (1)

$$A + B + C = 2$$

Since  $A = 4$

$$4 + \left[ \frac{-11 + 2C}{3} \right] + C = 2$$

$$4 - \frac{11 + 2C}{3} + C = 2$$

Multiply by 3

$$12 - 11 + 2C + 3C = 6$$

$$12 - 11 + 5C = 6$$

$$1 + 5C = 6$$

$$5C = 6 - 1$$

$$6C = 5$$

$$C = 1$$

Put  $C = 1$  in eqn (1)

$$4 + B + 1 = 2$$

$$5 + B = 2$$

$$B = 2 - 5$$

$$B = -3$$

$$\int \frac{4 dx}{x+1} + \int \frac{-3 dx}{x-2} + \int \frac{1 dx}{x+3} = \frac{2x^2 + 9x - 35}{(x+1)(x-2)(x+3)}$$

$$\text{Let } u = x + 1$$

$$du = dx$$

$$\int dx = \int du$$

$$4 \ln u$$

$$\text{let } u = x - 2$$

$$du = dx$$

$$\int dx = \int du$$

$$-3 \ln u$$

$$\text{let } u = x + 3$$

$$du = dx$$

$$\int dx = \int du$$

$$\ln u$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)$$