

MAT104

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$$1) \int \frac{11-3x}{x^2+2x-3} dx$$

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

$$11-3x = A(x+3) + B(x-1)$$

$$\text{If } x=1; 11-3(1) = A(1+3) + B(1-1)$$

$$\Rightarrow 8 = 4A$$

$$\Rightarrow A=2$$

$$\text{If } x=-3; 11-3(-3) = A(-3+3) + B(-3-1)$$

$$\Rightarrow 20 = -4B \Rightarrow B = -5$$

$$\therefore \int \frac{2 dx}{x-1} + \int \frac{-5 dx}{x+3} = \int \frac{11-3x}{x^2+2x-3} dx$$

$$\text{let } u = x-1$$

$$du = 1 dx$$

$$dx = du$$

$$\Rightarrow \int \frac{2 du}{u}$$

$$\Rightarrow 2 \ln(x-1)$$

$$\text{let } u = x+3$$

$$du = 1 dx$$

$$dx = du$$

$$\int \frac{-5 du}{u}$$

$$\Rightarrow -5 \ln(x+3)$$

$$\Rightarrow 2 \ln(x-1) - 5 \ln(x+3)$$

$$2) \int \frac{4x-16}{x^2-2x-3} dx$$

$$\frac{4x-16}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3} = \frac{A(x-3)+B(x+1)}{(x+1)(x-3)}$$

$$\therefore 4x-16 = A(x-3) + B(x+1)$$

$$\text{At } x=3; 4(3)-16 = A(3-3) + B(3+1)$$

$$\Rightarrow -4 = 4B \Rightarrow B = -1$$

$$\text{At } x=-1; 4(-1)-16 = A(-1-3) + B(-1+1)$$

$$\Rightarrow -20 = -4A \Rightarrow A = 5$$

$$\Rightarrow \int \frac{5}{x+1} dx + \int \frac{-4}{x-3} dx = \int \frac{4x-16}{x^2-2x-3} dx$$

let $u = x+1$	let $u = x-3$
$du = dx$	$du = dx$

$\Rightarrow \int \frac{5 du}{u}$	$\Rightarrow \int \frac{-4 du}{u}$
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$5 \ln(x+1)$	$\Rightarrow -4 \ln(x-3)$
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$$\Rightarrow 5 \ln(x+1) - 4 \ln(x-3)$$

$$3) \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$2x^2 - 9x - 35 = A(x+3)(x-2) + B(x+1)(x+3) + C(x+1)(x-2)$$

at  $x=2$  ✗

$$\Rightarrow 2(4) - 9(2) - 35 = B(3)(5) \neq$$

$$8 - 18 - 35 = 15B$$

$$-45 = 15B \Rightarrow B = -3$$

at  $x=-1$  ✗

$$\Rightarrow 2(1) - 9(-1) - 35 = A(2)(-3)$$

$$\Rightarrow 2 + 9 - 35 = -6A$$

$$-24 = -6A \Rightarrow A = 4$$

at  $x=-3$

$$\Rightarrow 2(-3) - 9(-3) - 35 = C(-2)(-5)$$

$$18 + 27 - 35 = 10C$$

$$10 = 10C \Rightarrow C = 1$$

$$\therefore \int \frac{4 dx}{x+1} + \int \frac{-3 dx}{x-2} + \int \frac{dx}{x+3}$$

$$\Rightarrow dx = du$$

$$\Rightarrow \int \frac{4 du}{x+1} + \int \frac{-3 du}{x-2} + \int \frac{du}{x+3}$$

$$\Rightarrow 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)$$