

MAT 104 ASSIGNMENT

① $\int \frac{11-3x}{(x^2+2x-3)} dx$

Solution:

$$\int \frac{11-3x}{(x-1)(x+3)} dx = \int \frac{A}{(x-1)} + \frac{B}{(x+3)} dx$$

$$11-3x = A(x+3) + B(x-1)$$

When $x = -3$

$$11-3(-3) = A(-3+3) + B(-3-1)$$

$$11+9 = A(0) + B(-4)$$

$$\frac{20}{-4} = \frac{-4B}{-4}$$

$$B = -5$$

When $x = 1$

$$11-3(1) = A(1+3) + B(1-1)$$

$$11-3 = A(4) + B(0)$$

$$\frac{8}{4} = \frac{4A}{4}$$

$$A = 2$$

$$\int \frac{A}{x-1} + \frac{B}{x+3} dx = \int \frac{2}{(x-1)} - \frac{5}{(x+3)} dx$$

$$\Rightarrow \cancel{2 \ln(x+3)} \quad 2 \ln(x-1) - 5 \ln(x+3) + C$$

$$\textcircled{2} \int \frac{4x-16}{(x^2-2x-3)} dx$$

Solution

$$\int \frac{4x-16}{(x+1)(x-3)} = \int \frac{A}{(x+1)} + \frac{B}{(x-3)} dx$$

$$4x-16 = A(x-3) + B(x+1)$$

When $x=3$

$$4(3)-16 = A(3-3) + B(3+1)$$

$$-4 = A(0) + B(4)$$

$$\frac{-4}{4} = \frac{4B}{4}$$

$$B = -1$$

When $x=-1$

$$4(-1)-16 = A(-1-3) + B(-1+1)$$

$$-4-16 = A(-4) + B(0)$$

$$\frac{-20}{-4} = \frac{-4A}{-4}$$

$$A = 5$$

$$\int \frac{A}{(x+1)} + \frac{B}{(x-3)} dx = \int \frac{5}{(x+1)} + \frac{-1}{(x-3)} dx$$

$$= 5 \ln(x+1) - \ln(x-3) + C$$

$$\textcircled{2} \int \frac{(2x^2 - 9x - 35)}{(x+1)(x+2)(x+3)} dx$$

Solution

$$\int \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)} dx$$

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x-1)(x+3) + C(x-1)(x-2)$$

When $x=1$

$$2(1)^2 - 9(1) - 35 = A(1-2)(1+3) + B(1-1)(1+3) + C(1-1)(1-2)$$

$$2 - 9 - 35 = A(-4) + B(0) + C(0)$$

$$\frac{-42}{-4} = \frac{-4A}{-4}$$

$$A = \frac{21}{2} \text{ or } 10.5$$

When $x=2$

$$2(2)^2 - 9(2) - 35 = A(2-2)(2+3) + B(2-1)(2+3) + C(2-1)(2-2)$$

$$8 - 18 - 35 = A(0) + B(5) + C(0)$$

$$\frac{-45}{5} = \frac{5B}{5}$$

$$B = -9$$

When $x=-3$

$$2(-3)^2 - 9(-3) - 35 = A(-3-2)(-3+3) + B(-3-1)(-3+3) + C(-3-1)(-3-2)$$

$$-18 + 27 - 35 = A(0) + B(0) + C(20)$$

$$\frac{10}{20} = \frac{20C}{20}$$

$$C = \frac{1}{2} \text{ or } 0.5$$

$$\int \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)} dx = \int \frac{21}{(2x-2)} + \frac{9}{(x-2)} + \frac{1}{(2x+6)} dx$$

$$\Rightarrow \frac{21}{2} \ln(2x-2) - 9 \ln(x-2) + \frac{1}{2} \ln(2x+6) + C$$