

$$1) \int \frac{11-3x}{\sqrt{x^2+2x-3}} dx$$

$$\begin{aligned} \frac{11-3x}{x^2+2x-3} &= \frac{11-3x}{(x-1)(x+3)} \\ &= \frac{A}{x-1} + \frac{B}{x+3} \\ &= \frac{A(x+3) + B(x-1)}{(x-1)(x+3)} \end{aligned}$$

$$A(x+3) + B(x-1) = 11-3x$$

$$Ax+3A+Bx-B = 11-3x$$

$$(A+B)x + (3A-B) = 11-3x$$

$$A+B = -3 \quad \text{--- (i)}$$

$$3A-B = 11 \quad \text{--- (ii)}$$

$$4A = 8$$

$$A = 8/4 = 2$$

Substitute  $A=2$  in eqn (i)

$$A+B = -3$$

$$2+B = -3$$

$$B = -5$$

$$2) \int \frac{4x-16}{\sqrt{x^2-2x-3}} dx$$

$$\begin{aligned} \frac{4x-16}{x^2-2x-3} &= \frac{4x-16}{(x+1)(x-3)} \\ &= \frac{A}{x+1} + \frac{B}{x-3} \\ &= \frac{A(x-3) + B(x+1)}{(x+1)(x-3)} \end{aligned}$$

$$A(x-3) + B(x+1) = 4x-16$$

$$Ax-3A+Bx+B = 4x-16$$

$$(A+B)x + (-3A+B) = 4x-16$$

$$A+B = 4 \quad \text{--- (i)}$$

$$-3A+B = -16 \quad \text{--- (ii)}$$

$$4A = 20$$

$$A = 20/4 = 5$$

Substitute  $A=5$  in eqn (i)

$$A+B = 4$$

$$5+B = 4$$

$$B = -1$$

$$3) \int \frac{(2x^2-9x-35)}{\sqrt{(x+1)(x-2)(x+3)}} dx$$

$$3) \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

f(2)

$$2(2^2) - 9(2) - 35 = A(2-2)(2+3) + B(2+1)(2+3) + C(2+1)(2-2)$$

$$8 - 18 - 35 = A(0)(5) + B(3)(5) + C(3)(0)$$

$$-45 = 15B$$

$$B = -3 //$$

f(-3)

$$2(-3)^2 - 9(-3) - 35 = A(-3-2)(-3+3) + B(-3+1)(-3+3) + C(-3+1)(-3-2)$$

$$18 + 27 - 35 = A(-5)(0) + B(-2)(0) + C(-2)(-5)$$

$$10 = 10C$$

$$C = 1 //$$

f(-1)

$$2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3) + B(-1+1)(-1+3) + C(-1+1)(-1-2)$$

$$2 + 9 - 35 = A(-3)(2) + B(0)(2) + C(0)(-3)$$

$$-24 = -6A$$

$$A = 4 //$$

$$A = 4, B = -3, C = 1$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} dx + \int \frac{-3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$\text{Let } u = x+1 \quad du/dx = 1 \quad dx = du$$

$$\int \frac{4}{x+1} dx = \int \frac{4 \cdot du}{u} = 4 \int \frac{1}{u} du = 4 \ln u = 4 \ln(x+1)$$

$$\text{Let } u = x-2 \quad du/dx = 1 \quad dx = du$$

$$\int \frac{-3}{x-2} dx = \int \frac{-3 \cdot du}{u} = -3 \int \frac{1}{u} du = -3 \ln u = -3 \ln(x-2)$$

$$\text{Let } u = x+3 \quad du/dx = 1 \quad dx = du$$

$$\int \frac{1}{x+3} dx = \int \frac{1}{u} du = \ln u = \ln(x+3)$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)$$