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Matric no: [illegible]
Department: Medicine and Surgery
Course: Mat 164

1. $\int \frac{11-3x}{(x^2+2x-3)} dx$

2. $\int \frac{4x-16}{(x^2-2x-3)} dx$

3. $\int \frac{(2x^2-9x-35)}{(x+1)(x-2)(x+3)} dx$

Solution

1. $\int \frac{11-3x}{(x^2+2x-3)} dx$

But $(x^2+2x-3) = (x-1)(x+3)$

$$\int \frac{11-3x}{(x-1)(x-3)} dx = \int \frac{A}{x-1} + \frac{B}{x-3} dx$$

$$\therefore \int \frac{11-3x}{(x-1)(x-3)} dx = \int \frac{A}{x-1} + \frac{B}{x+3} dx$$

$$11-3x = A(x+3) + B(x-1)$$

~~$4-3x$~~ Put $x=1$ into the equation

$$11-3(1) = A((1)+3) + B(1-1)$$

$$8 = 4A$$

$$\therefore A = 2$$

Put $x=-3$ into the eqn

$$A(-3)+3) + B(-3)-1) = 11-3(-3)$$

$$-4B = 20$$

$$\therefore B = -5$$

$$\frac{2}{(x-1)} - \frac{5}{(x+3)} = \frac{11-3x}{(x-1)(x+3)}$$

$$\int \frac{11-3x}{(x-1)(x+3)}\,dx = \int \frac{2}{(x-1)}\,dx - \int \frac{5}{(x+3)}\,dx$$

$$\int \frac{11-3x}{(x-1)(x+3)}\,dx = 2\ln(x-1) + 5\ln(x+3) + C$$

2. $\int \frac{4x-16}{(x^2-2x-3)}\,dx$

But $(x^2-2x-3) = (x+1)(x-3)$

$$\frac{4x-16}{(x^2-2x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$4x-16 = A(x-3) + B(x+1)$$

Put $x = -1$ into the equation

$$4(-1)-16 = A(-1-3) + B(-1+1)$$

$$-21 = -4A$$

$$A = 5\tfrac{1}{4}$$

Put $x = 3$ into the equation

$$4(3)-16 = A(3-3) + B(3+1)$$

$$-4 = 4B$$

$$B = -1$$

$$\int \frac{4x-16}{(x^2-2x-3)}\,dx = \frac{21}{4}\int \frac{1}{(x+1)}\,dx - \int \frac{1}{(x-3)}\,dx$$

$$\int \frac{4x-16}{(x^2-2x-3)}\,dx = \tfrac{21}{4}\ln(x+1) - 1\ln(x-3) + C$$

3. $\frac{2x^2-9x-35}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$

$2x^2-9x-35 = A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2)$

$2x^2-9x-35 = A(x^2+5x+6) + B(x^2+4x+3) + C(x^2+3x+2)$

Put $x=-1$ into the equation.

$-24 = 2A$

$A = -12$

Put $x=-2$ into the equation.

$-9 = 1B$

$B = -3/5$

Put $x=-3$ into the equation.

$10 = 2C$

$C = 5$

$\frac{2x^2-9x-35}{(x+1)(x+2)(x+3)} = \frac{-12}{x+1} - \frac{3/5}{x+2} + \frac{5}{x+3}$

$\int \frac{2x^2-9x-35}{(x+1)(x+2)(x+3)}\,dx = -12\int \frac{1}{x+1}\,dx - \frac{3}{5}\int \frac{1}{x+2}\,dx + 5\int \frac{1}{x+3}\,dx$

$\int \frac{2x^2-9x-35}{(x+1)(x+2)(x+3)}\,dx = -12\ln(x+1) - 3/5\ln(x+2) + 5\ln(x+3) + C$