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$$\int \frac{11-3x}{x^2+2x-3} dx$$

$$x^2 + 2x - 3 = (x+3)(x-1)$$

$$\frac{11-3x}{x^2+2x-3} = \frac{A}{x+3} + \frac{B}{x-1}$$

$$\frac{11-3x}{x^2+2x-3} = \frac{A(x-1) + B(x+3)}{(x+3)(x-1)}$$

$$11-3x = A(x-1) + B(x+3)$$

$$f(-3) \Rightarrow 11-3(-3) = A(-3-1)$$

$$\frac{20}{-4} = \frac{-4A}{-4}$$

$$A = -5 //$$

$$f(1) \Rightarrow 11-3(1) = B(1+3)$$

$$8 = 4B$$

$$B = 2 //$$

$$\int \frac{11-3x}{x^2+2x-3} dx = \int \frac{-5}{x+3} dx + \int \frac{2}{x-1} dx$$

$$\int \frac{-5}{x+3} dx \Rightarrow \text{let } u = x+3$$

$$\frac{du}{dx} = 1 \quad dx = du$$

$$= \int \frac{-5}{u} \cdot du \Rightarrow -5 \int \frac{1}{u} du$$

$$= -5 \ln u$$

$$= -5 \ln(x+3)$$

$$\int \frac{2}{x-1} dx \Rightarrow \text{let } u = x-1$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$\int \frac{2}{x-1} dx = \int \frac{2}{u} \cdot du \Rightarrow 2 \int \frac{1}{u} \cdot du$$

$$= 2 \ln u = 2 \ln(x-1)$$

$$\int \frac{11-3x}{x^2+2x-3} = 2 \ln(x-1) - 5 \ln(x+3) + C$$

$$2 \int \frac{4x-16}{x^2-2x-3} dx$$

$$x^2-2x-3 = (x-3)(x+1)$$

$$\frac{4x-16}{x^2-2x-3} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$\frac{4x-16}{x^2-2x-3} = \frac{A(x+1) + B(x-3)}{(x-3)(x+1)}$$

$$4x-16 = A(x+1) + B(x-3)$$

$$f(3) \Rightarrow 4(3)-16 = A(3+1)$$

$$12-16 = 4A$$

$$-4 = 4A$$

$$A = -1$$

$$f(-1) \Rightarrow 4(-1)-16 = B(-1-3)$$

$$-4-16 = -4B$$

$$-20 = -4B$$

$$B = 5$$

$$\int \frac{4x-16}{x^2-2x-3} dx = \int \frac{-1}{x-3} dx + \int \frac{5}{x+1} dx$$

$$\int \frac{-1}{x-3} dx \Rightarrow \text{let } u = x-3$$

$$\frac{du}{dx} = 1$$

$$dx = du$$

$$\int \frac{-1}{x-3} dx = \int \frac{-1}{u} du = -1 \ln u \\ = -\ln(x-3)$$

$$\int \frac{5}{x+1} dx \implies \text{let } u = x+1 \\ du = dx$$

$$\int \frac{5}{x+1} dx = \int \frac{5}{u} du \implies 5 \int \frac{1}{u} du \\ 5 \ln u = 5 \ln(x+1)$$

$$\int \frac{4x-16}{x^2-2x-3} dx = -\ln(x-3) + 5 \ln(x+1) + C$$

$$\int \frac{4x-16}{x^2-2x-3} dx = 5 \ln(x+1) - \ln(x-3) + C$$

$$3 \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx$$

$$= \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$2x^2 - 9x - 35 = A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2)$$

$$2x^2 - 9x - 35 = Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$2x^2 - 9x - 35 = (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C)$$

$$A+B+C = 2 \quad \text{--- (1)}$$

$$A+4B-C = -9 \quad \text{--- (2)}$$

$$(-6A+3B-2C) = -35 \quad \text{--- (3)}$$

$$\text{From (1), } A = 2 - B - C \quad \text{--- (4)}$$

From (2)

$$2 - B - C + 4B - C = -9$$

$$4B - B - C - C = -9 - 2$$

$$3B - 2C = -11 \quad \text{--- (5)}$$

From (3)

$$-6(2 - B - C) + 3B - 2C = -35$$

$$-12 + 6B + 6C + 3B - 2C = -35$$

$$6B + 3B + 6C - 2C = -35 + 12$$

$$9B + 4C = -23 \quad \text{--- (6)}$$

$$\begin{aligned} \text{Eq. 5-6: } 9 \times 3B - 2C &= -11 \\ 3 \times 9B + 4C &= -23 \end{aligned}$$

$$\begin{aligned} 27B - 18C &= -99 \\ -27B + 12C &= -69 \\ -30C &= -30 \end{aligned}$$

$$C = 1 //$$

$$\text{Also, } 27B - 18(1) = -99$$

2c)

$$27B - 18 = -99$$

$$27B = -99 + 18$$

$$27B = -81$$

$$B = -3 //$$

$$\text{Finally, } A + B + C = 2$$

$$A - 3 + 1 = 2$$

$$A - 2 = 2$$

$$A = 4 //$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} dx + \int \frac{-3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$\int \frac{4}{x+1} dx, \text{ let } u = x+1 \quad \frac{du}{dx} = 1 \quad du = dx$$

$$\int \frac{4}{u} \cdot du = 4 \ln u = 4 \ln(x+1)$$

$$\int \frac{-3}{x-2} dx = -3 \ln 4 - 3 \ln(x-2)$$

$$\int \frac{1}{x+3} dx = \ln(x+3)$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3) + C$$