(i) Equation of the tangent:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x - 1)$$

$$7y - 14 = -8(x - 1)$$

$$7y - 14 = -8x + 8$$

$$7y = -8x + 22$$

Hence, $7y + 8x - 22 = 0$ is the equation of the tangent

(ii) Equation of the normal:

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - 2 = \frac{-1}{-8/7}(x - 1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y - 7x - 9 = 0$$

Hence, $8y - 7x - 9 = 0$ is the equation of the normal

$m_2 = 1$

$m_1 m_2 = -1$

$\frac{2}{3} \times 1 = \frac{2}{3} \neq -1$

Hence, the lines are NOT PERPENDICULAR

③ Find the equations of the tangent and normal to the axis $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$

Solution

$$x^2 + y^2 + 3xy - 11 = 0$$

$$\frac{dy}{dx} \Big| \quad 2x + 2y\frac{dy}{dx} + 3y + 3x\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} + 3x\frac{dy}{dx} = -3y - 2x$$

$$\frac{dy}{dx}(2y + 3x) = -3y - 2x$$

$$\frac{dy}{dx} = \frac{-3y - 2x}{2y + 3x}$$

$$m = \frac{dy}{dx}\Big|_{x=1, y=2} = \frac{-3(2) - 2(1)}{2(2) + 3(1)} = \frac{-6 - 2}{4 + 3} = \frac{-8}{7}$$

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Assignment

Equation of tangent and normal

For the curves in Problem 1 to 5, at the points given, find (a) the equation of the tangent and b) the equation of the normal

① $y = 2x^2$ at the point (1, 2)

Solution

$y = 2x^2$

$dy/dx = 4x$

$m = 4 \times 1$

$m = 1$

$x_1 = 1, y_1 = 2$

$y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y = 4x - 4 + 2$

$y = 4x - 2$

$\therefore y = 4x - 2$ gives the equation of the tangent

(b) Equation of the normal

$M_1 M_2 = -1$

$3 \times \frac{-1}{3} = -1$

Hence, lines $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular

② $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Solution

$3y - 4 = 2x + 3$

$y - 5 = x + 6$

For the lines to be perpendicular, then $m_1 m_2 = -1$

$3y - 4 = 2x + 3$

$3y = 2x + 7$

$y = \frac{2}{3}x + \frac{7}{3}$

By comparison to $y = mx + c$,

$m_1 = \frac{2}{3}$

$y - 5 = x + 6$

$y = x + 11$

By comparison to $y = mx + c$,