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 19THMS 01/300
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 Mat 104

1. Evaluate: $\int \frac{11-3x}{x^2+2x-3} dx$

Solution

$\int \frac{11-3x}{x^2+2x-3} dx$

$\Rightarrow \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$
 $= \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$

$\Rightarrow A(x+3) + B(x-1) = 11-3x$

$\Rightarrow Ax + 3A + Bx - B = 11-3x$

$\Rightarrow Ax + Bx + 3A - B = 11-3x$

$\Rightarrow (A+B)x + (3A-B) = 11-3x$

$\Rightarrow A+B = -3 \quad \dots \dots (i) \times 3$

$3A-B = 11 \quad \dots \dots (ii) \times 1$

$3A+3B = -9 \quad \dots \dots (iii)$

$-3A-B = 11 \quad \dots \dots (iv)$

* $4B = -20$

$\therefore B = -5$

Also: From equ (i)

$A+B = -3$

$\Rightarrow A = -3-B$

$\Rightarrow A = -3-(-5)$

$\Rightarrow A = 2$

$\Rightarrow \int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx = \int \frac{11-3x}{x^2+2x-3} dx$

$\Rightarrow \int \frac{2}{x-1} dx - \int \frac{5}{x+3} dx = \int \frac{11-3x}{x^2+2x-3} dx$

let $u = x-1$ $u = x+3$

$\frac{du}{dx} = 1$ $\frac{du}{dx} = 1$

$dx = du$ $dx = du$

$\frac{dx}{du} = 1$ $\frac{dx}{du} = 1$

du du

$\Rightarrow 2 \int \frac{du}{u}$ $\Rightarrow 5 \int \frac{du}{u}$

$\Rightarrow 2 \ln u$ $= 5 \ln u$

$\Rightarrow 2 \ln(x-1) - 5 \ln(x+3)$

$\Rightarrow \int \frac{11-3x}{x^2+2x-3} = 2 \ln(x-1) - 5 \ln(x+3)$

2. Simplify: $\int \frac{4x-16}{x^2-2x} dx$

Solution

$$\int \frac{4x-16}{x^2-2x} dx$$

$$\Rightarrow \frac{4x-16}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}$$

$$= \frac{A(x-2) + Bx}{x(x-2)}$$

$$\Rightarrow A(x-2) + Bx = 4x-16$$

$$\Rightarrow Ax - 2A + Bx = 4x - 16$$

$$\Rightarrow Ax + Bx - 2A = 4x - 16$$

$$\Rightarrow (A+B)x - 2A = 4x - 16$$

$$\Rightarrow A+B = 4 \quad \dots (i) \times 2$$

$$-2A = -16 \quad \dots (ii) \times 1$$

$$2A + 2B = 8 \quad \dots (iii)$$

$$+ 2A = -16 \quad \dots (iv)$$

$$* 2B = -8$$

$$\Rightarrow B = -4$$

Also: from eqn (i)

$$A+B=4$$

$$A=4-B$$

$$\Rightarrow A=4-(-4)$$

$$A=8$$

$$\Rightarrow \int \frac{8}{x} dx + \int \frac{-4}{x-2} dx = \int \frac{4x-16}{x^2-2x} dx$$

$$\Rightarrow \int \frac{8}{x} dx - \int \frac{4}{x-2} dx = \int \frac{4x-16}{x^2-2x} dx$$

Let $u=x$

$u=x-2$

$$\frac{du}{dx} = 1$$

$$\frac{du}{dx} = 1$$

$$dx$$

$$dx$$

$$\frac{dx}{du} = 1 \quad \frac{dx}{du} = 1$$

$$\Rightarrow \int \frac{8}{u} du \quad 4 \int \frac{du}{u}$$

$$\Rightarrow 8 \ln u \quad 4 \ln u$$

$$\Rightarrow 8 \ln(x) - 4 \ln(x-2)$$

$$\Rightarrow \int \frac{4x-16}{x^2-2x} = 8 \ln(x) - 4 \ln(x-2)$$

$$3 \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}$$

$$\Rightarrow \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\Rightarrow 2x^2 - 9x - 35 = A(x+3)(x-2) + B(x+1)(x+3) + C(x+1)(x-2)$$

\therefore Using $x = -1$ or $x = 2$ or $x = -3$

$$\Rightarrow f(-1) = 2(-1)^2 - 9(-1) - 35 = A(-1+3)(-1-2)$$

$$= 2 + 9 - 35 = A(2)(-3)$$

$$\Rightarrow -24 = -6A$$

$$= A = 4$$

$$f(2) \Rightarrow 2(2)^2 - 9(2) - 35 = B(2+1)(2+3)$$

$$= 8 - 18 - 35 = B(3)(5)$$

$$-45 = 15B$$

$$\Rightarrow B = -3$$

$$f(-3) \Rightarrow 2(-3)^2 - 9(-3) - 35 = C(-3+1)(-3-2)$$

$$\Rightarrow 18 + 27 - 35 = C(-2)(-5)$$

$$= 10 = 10C$$

$$\Rightarrow C = 1$$

$$\Rightarrow 4 \int \frac{dx}{x+1} - 3 \int \frac{dx}{x-2} + \int \frac{dx}{x+3} = \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx$$

$$\text{let } u = x+1 \quad \text{let } u = x-2 \quad \text{let } u = x+3$$

$$du = dx$$

$$du = dx$$

$$du = dx$$

$$4 \ln u$$

$$\Rightarrow -3 \ln u$$

$$= \ln u$$

$$\Rightarrow 4 \ln(x+1)$$

$$\Rightarrow -3 \ln(x-2)$$

$$= \ln(x+3)$$

$$\Rightarrow \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)$$