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1)
$$\frac{11-3x}{x^2+2x-3}$$

$$\Rightarrow \int \frac{11-3x}{x^2+2x-3}\,dx$$

from the denominator,

$$x^2+2x-3=0$$

$$(x+3)(x-1)=0$$

$$\therefore \int \frac{11-3x}{x^2+2x-3}\,dx = \int \frac{11-3x}{(x+3)(x-1)}\,dx$$

Resolving $\frac{11-3x}{(x+3)(x-1)}$ into partial fraction

$$\frac{11-3x}{(x+3)(x-1)} = \frac{A}{(x+3)} + \frac{B}{(x-1)}$$

$$\frac{11-3x}{(x+3)(x-1)} = \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}$$

Equating the numerators

$$11-3x = A(x-1) + B(x+3)$$

Put $x=1$

$$11-3(1) = A(1-1) + B(1+3)$$

$$8 = 4B$$

$$B = 2$$

Put $x=-3$

$$11-3(-3) = A(-3-1) + B(-3+3)$$

$$20 = -4A$$

$$A = 5$$

$$\therefore \frac{11-3x}{(x+3)(x-1)} = \frac{-5}{(x+3)} + \frac{2}{(x-1)}$$

$$\int \left(\frac{11-3x}{(x+3)(x-1)}\right)dx = \int\left(\frac{-5}{(x+3)} + \frac{2}{(x-1)}\right) dx =$$

$$\int \frac{-5}{(x+3)}\,dx + \int \frac{2}{(x-1)}\,dx = -5\int \frac{1}{(x+3)}\,dx + 2\int \frac{1}{(x-1)}\,dx =$$

$$-5\ln(x+3) + 2\ln(x-1)$$

2) $$\int \frac{4x-16}{x^2-2x-3}$$

From the denominator,

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$$\therefore \int \frac{4x-16}{x^2-2x-3} = \int \frac{4x-16}{(x+1)(x-3)}$$

Resolving $\frac{4x-16}{(x+1)(x-3)}$ into partial fraction

$$\frac{4x-16}{(x+1)(x-3)} = \frac{A}{(x-3)} + \frac{B}{(x+1)}$$

$$\frac{4x-16}{(x+1)(x-3)} = \frac{A(x+1) + B(x-3)}{(x+1)(x-3)}$$

Equating the numerators

$4x - 16 = A(x+1) + B(x-3)$

Put $x = 3$

$4(3) - 16 = A(3+1) + B(3-3)$

$-4 = 4A$

$A = -1$

Put $x = -1$

$4(-1) - 16 = A(-1+1) + B(-1-3)$

$-20 = -4B$

$B = 5$

$$\therefore \frac{4x-16}{(x-3)(x+1)} = \frac{-1}{(x-3)} + \frac{5}{(x+1)}$$

$$\int\left(\frac{4x-16}{(x-3)(x+1)}\right)dx = \int\left(\frac{-1}{(x-3)} + \frac{5}{(x+1)}\right)dx = \int\frac{-1}{(x-3)}\,dx + 5\int\frac{1}{(x+1)}\,dx =$$

$$\Rightarrow -1\ln(x-3) + 5\ln(x+1)$$

3) $$\int\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$$

Resolving into partial fraction

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+1)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

Equating the numerators

$$2x^2-9x-35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

Put $x = 2$

$-45 = 15B$

$B = -3$

Put $x = -1$

$-24 = -6A$

$A = 4$

Put $x = -3$

$10 = 10C$

$C = 1$

$$\therefore \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{4}{(x+1)} - \frac{3}{(x-2)} + \frac{1}{(x+3)}$$

$$\int\left(\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\right)dx = \int\left(\frac{4}{(x+1)} - \frac{3}{(x-2)} + \frac{1}{(x+3)}\right)dx$$

$$\int\frac{4}{(x+1)}\,dx - \int\frac{3}{(x-2)}\,dx + \int\frac{1}{(x+3)}\,dx$$

$$= 4\int\frac{1}{(x+1)}\,dx - 3\int\frac{1}{(x-2)}\,dx + \int\frac{1}{(x+3)}\,dx =$$

$$\Rightarrow 4\ln(x+1) - 3\ln(x-2) + \ln(x+3).$$