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19/MHS01/102 MAT104 MBBS

Question 1

$$\int \frac{11-3x}{x^2+2x-3} dx$$

$$\Rightarrow \frac{11-3x}{(x-1)(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)}$$

multiply both sides by $(x-1)(x+3)$

$$\Rightarrow 11-3x = A(x+3) + B(x-1)$$

when $x = -3$

$$11-3(-3) = A(-3+3) + B(-3-1)$$

$$20 = -4B \therefore B = -5$$

when $x = +1$

$$11-3(1) = A(1+3) + B(1-1)$$

$$8 = 4A \therefore A = 2$$

$$\Rightarrow \frac{2}{x+1} + \frac{-5}{x+3}$$

$$\int \frac{2}{x+1} dx + \int \frac{-5}{x+3} dx, \text{ since } \int \frac{A}{ax+b} dx = \frac{A \ln|ax+b|}{a} + C$$

$$\Rightarrow 2 \ln|x+1| - 5 \ln|x+3| + C$$

$$\int \frac{11-3x}{x^2+2x-3} = 2 \ln|x+1| - 5 \ln|x+3| + C$$

Question 2

$$\int \frac{4x-16}{x^2-2x-3} dx \Rightarrow \frac{4x-16}{(x-3)(x+1)} \text{ as partial fraction}$$

$$\frac{4x-16}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)}$$

multiply both sides by $(x-3)(x+1)$

$$4x-16 = A(x+1) + B(x-3)$$

when $x = -1$

$$4(-1)-16 = 0 + -4B$$

$$-20 = -4B \quad B = 5$$

when $x = +3$

$$4(3)-16 = 4A$$

$$-4 = 4A \quad A = -1$$

$$\therefore \frac{-1}{(x-3)} + \frac{5}{(x+1)}$$

$$\int \frac{-1}{(x-3)} dx + \int \frac{5}{(x+1)} dx$$

$$-1 \ln|x-3| + 5 \ln|x+1| + C$$

$$\int \frac{4x-16}{(x-3)(x+1)} dx = \int \frac{4x-16}{x^2-2x-3} dx = 5 \ln|x+1| - \ln|x-3| + C$$

$$5 \ln|x+1| - \ln|x-3| + C //$$

Question 3

$$\int \frac{(2x^2 - 9x - 35) dx}{(x+1)(x-2)(x+3)}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

multiply By $(x+1)(x-2)(x+3)$

$$\Rightarrow 2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

When $x = -1$; $2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3)$
 $2 + 9 - 35 = A(-3)(2) \Rightarrow -24 = -6A$
 $A = 4$

When $x = 2$; $2(2)^2 - 9(2) - 35 = B(2+1)(2+3)$
 $-45 = 15B \therefore B = -3$

When $x = -3$; $2(-3)^2 - 9(-3) - 35 = C(-3+1)(-3-2)$
 $10 = 10C \therefore C = 1$

$$\Rightarrow \frac{4}{(x+1)} + \frac{-3}{(x-2)} + \frac{1}{(x+3)}$$

$$\int \frac{4}{x+1} dx - \int \frac{3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3| + C$$

$$\therefore \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = 4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3| + C$$