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MAT NO: 19/MHS01/274

DEPT: MEDICINE AND SURGERY

MAT 104 ASSIGNMENT

Question

1. $\int \frac{11-3x}{x^2+2x-3}\,dx$

Solution:

Expressing $\frac{11-3x}{x^2+2x-3}$ as a partial fraction;

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$11-3x = A(x+3) + B(x-1)$$

where $x+3=0$, $x=-3$

$$11-3(-3) = A(-3+3) + B(-3-1)$$

$$11+9 = 0-4B$$

$$\frac{4B}{4} = \frac{-20}{4}$$

$$B = -5$$

where $x-1=0$ $x=1$

$$11-3(1) = A(1+3) + B(1-1)$$

$$8 = 4A + 0$$

$$\frac{4A}{4} = \frac{8}{4}$$

$$A = 2$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{2}{x-1} + \left(\frac{-5}{x+3}\right)$$

$$\int \frac{11-3x}{(x-1)(x+3)}\,dx = \int \frac{2}{x-1}\,dx + \int \frac{-5}{x+3}\,dx$$

since $\int \frac{A}{ax+b}\,dx = A\,\frac{\ln(ax+b)}{a} + c$

$$\therefore \int \frac{11-3x}{(x-1)(x+3)}\,dx = 2\ln(x-1) - 5\ln(x+3)$$

2. $\int \frac{4x-16}{x^2-2x-3}\,dx$

Resolving $\frac{4x-16}{x^2-2x-3}$ into partial fractions.

$$\frac{4x-16}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3}$$

$4x-16 = A(x-3) + B(x+1)$

where $x-3=0 \quad x=3$

$4(3)-16 = A(3-3) + B(3+1)$

$-4 = 0 + 4B$

$\frac{4B}{4} = \frac{-4}{4}$

$B = -1$

where $x+1=0 \quad x=-1$

$4(-1)-16 = A(-1-3) + B(-1+1)$

$-20 = -4A + 0$

$\frac{-4A}{-4} = \frac{-20}{-4}$

$A = 5$

$$\frac{4x-16}{x^2-2x-3} = \frac{5}{x+1} + \frac{(-1)}{x-3}$$

$$\int \frac{4x-16}{x^2-2x-3}\,dx = \int \frac{5}{x+1}\,dx + \int \frac{-1}{x-3}\,dx$$

$$\therefore \int \frac{4x-16}{x^2-2x-3}\,dx = 5\ln(x+1) - \ln(x-3)$$

3. $\int \frac{(2x^2-9x-35)}{(x+1)(x-2)(x+3)}\,dx$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{x+3}$$

$$2x^2-9x-35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

when $x+1=0 \quad x=-1$

$$2(-1)^2-9(-1)-35 = A(-1-2)(-1+3)+B(-1+1)(-1+3)+C(-1+1)(-1-2)$$

$$2(1)+9-35 = A(-3)(2)+0+0$$

$$\frac{-24}{-6} = \frac{-6A}{-6}$$

$$A = 4$$

when $x-2=0 \quad x=2$

$$2(2)^2-9(2)-35 = A(2-2)(2+3)+B(2+1)(2+3)+C(2+1)(2-2)$$

$$2(4)-18-35 = 0+B(3)(5)+0$$

$$\frac{-45}{15} = \frac{15B}{15}$$

$$B = -3$$

when $x+3=0 \quad x=-3$

$$2(-3)^2-9(-3)-35 = A(-3-2)(-3+3)+B(-3+1)(-3+3)+C(-3+1)(-3-2)$$

$$2(9)+27-35 = 0+0+C(-2)(-5)$$

$$\frac{10}{10} = \frac{10C}{10}$$

$$C = 1$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} + \frac{(-3)}{x-2} + \frac{1}{x+3}$$

$$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx = \int \frac{4}{x+1}\,dx + \int \frac{-3}{x-2}\,dx + \int \frac{1}{x+3}\,dx$$

$$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3)$$