OKORIE GLORY OHUMASEFUNMI
MEDICINE AND SURGERY
19/MHS01/323
MAT 104 ASSIGNMENT

1) $\int \frac{11-3x}{x^2+2x-3}\,dx$

Solution.

$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)}$

$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)}$

Find the L.C.M of the RHS

$\frac{A}{(x-1)} + \frac{B}{(x+3)}$

$\frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$

$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$

Multiply both sides by $(x-1)(x+3)$

$11-3x = A(x+3) + B(x-1)$

When $x = -3$

$B(-3-1) = 11-3(-3)$

$B(-4) = 11+9$

$-4B = 20$

$B = -5$.

When $x = 1$

$A(1+3) = 11-3(1)$

$A(4) = 8$

$A = 2$.

To integrate

$\int \frac{2}{x-1}\,dx + \int \frac{-5}{x+3}\,dx = \int \frac{11-3x}{x^2+2x-3}\,dx$

Let $u = x-1$

$du = dx$

$dx = du$

$2\int \frac{du}{u}$

Let $u = x+3$

$du = dx$

$dx = du$

$-5\int \frac{du}{u}$

$\therefore \int \frac{11-3x}{x^2+2x-3}\,dx = 2\ln(x-1) - 5\ln(x+3)$

e) $\int \frac{4x-16}{x^2-2x-3}\,dx$

Solution.

$\frac{4x-16}{x^2-2x-3} = \frac{A}{(x+1)} + \frac{B}{(x-3)}$

Find the LCM of the RHS.

$\frac{A}{x+1} + \frac{B}{x-3}$

$\frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$

$\frac{4x-16}{(x+1)(x-3)} = \frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$

Multiply through by $(x+1)(x-3)$

$4x-16 = A(x-3) + B(x+1)$

When $x = 3$

$B(3+1) = 4(3) - 16$

$B(4) = 12 - 16$

$4B = -4$

$B = -1$ //.

When $x = -1$

$A(-1-3) = 4(-1) - 16$

$A(-4) = -20$

$-4A = 20$

$A = 5$ //.

Hence,

$$\int \frac{5}{x+1}\,dx + \int \frac{-1}{x-3}\,dx = \int \frac{4x-16}{x^2-2x-3}\,dx$$

Let $u = x+1$ | Let $u = x-3$

$du = dx$ | $du = dx$

$dx = du$ | $dx = du$

$$5\int \frac{du}{u} \qquad -1\int \frac{du}{du} = \int \frac{4x-16}{x^2-2x-3}\,dx$$

$$\therefore \int \frac{4x-16}{x^2-2x-3} = 5\ln u - \ln u$$

$$\int \frac{4x-16}{x^2-2x-3} = 5\ln(x+1) - \ln(x-3)$$

3) $$\int \frac{2x^2-9-35}{(x+1)(x-2)(x+3)}\,dx$$

$$\frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

Find the Lcm.

$$\frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x+3)}{(x+1)(x-2)(x+3)}$$

Multiply through by $(x+1)(x-2)(x+3)$

$$2x^2-9x-35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x+3)$$

$$2x^2-9x-35 = A(x^2+x-6) + B(x^2+4x+3) + C(x^2-x-2)$$

$$2x^2-9x-35 = Ax^2+Ax-6A+Bx^2+4Bx+3B+Cx^2-Cx-2C$$

$$2x^2-9x-35 = Ax^2+Bx^2+Cx^2+Ax+4Bx+Cx-6A+3B-2C$$

$$2x^2-9x-35 = x^2(A+B+C) + x(A+4B-C) + (-6A+3B-2C)$$

Compare the coefficient of both sides.

$$A+B+C = 2 \quad \text{---} \quad (I)$$

$$A+4B-C = -9 \quad \text{---} \quad (II)$$

$$-6A+3B-2C = -35 \quad \text{---} \quad (III)$$

From eqn (I), $A = 2-B-C$ ---- (IV)

Substitute eqn (iv) in equ (ii)

$2-B-C+4B-C=-9$

$3B-2C=-11$

$3B=-11+2C$ ——— (v)

Sub. equ (v) in equ (iii)

$-6A-11+2C-2C=-35$

$-6A-11=-35$

$-6A=-35+11$

$-6A=-24$

$A=4$ //.

Recall $3B=-11+2C$

$B=\frac{-11+2C}{3}$ ——— (vi)

Sub eqn (vi) in equ (i)

$A+B+C=2$

$4+\left(\frac{-11+2C}{3}\right)+C=2$

$4-\frac{11+2C}{3}+C=2$

Multiply both sides by 3

$12-11+2C+3C=6$

$12-11+5C=6$

$1+5C=6$

$5C=6-1$

$5C=5$

$C=1$ //.

Sub c in eq 1

$A+B+C=2$; $4+B+1=2$

$5B\ 5+B=2$

$B=2-5$

$B=-3$ //.

Hence.

$$\int\frac{4dx}{x+1}+\int\frac{-3dx}{x-2}+\int\frac{1dx}{x+3}=\int\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}dx$$

Let $u = x+1$ | Let $u = x-2$ | Let $u = x+3$

$du = dx$ | $du = dx$ | $du = dx$

$dx = du$ | $dx = du$ | $dx = du$.

$4 \ln u$ | $-3 \ln u$ | $\ln u$

$$\therefore \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}\, dx = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3)$$