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19/MHS01/358

MATH 104 ASSIGNMENT.

1. $\int \frac{11-3x}{x^2+2x-3} dx$

$$x^2+2x-3 = (x+3)(x-1)$$

$$\frac{11-3x}{x^2+2x-3} = \frac{A}{(x+3)} + \frac{B}{(x-1)}$$

$$\frac{11-3x}{x^2+2x-3} = \frac{A(x-1) + B(x+3)}{(x+3)(x-1)}$$

$$11-3x = A(x-1) + B(x+3)$$

$$f(-3) \Rightarrow 11-3(-3) = A(-3-1)$$

$$= \frac{20}{-4} = \frac{-4A}{-4}$$

$$A = -5$$

$$f(1) \Rightarrow 11-3(1) = B(1+3)$$

$$8 = 4B$$

$$\therefore B = 2$$

$$\int \frac{11-3x}{x^2+2x-3} dx = \int \frac{-5}{x+3} dx + \int \frac{2}{x-1} dx$$

$$\int \frac{-5}{x+3} dx \Rightarrow \text{let } u = x+3$$

$$\frac{du}{dx} = 1, dx = du$$

$$\int \frac{-5}{u} du \Rightarrow -5 \int \frac{1}{u} du$$

$$= -5 \ln u$$

$$\Rightarrow -5 \ln(x+3)$$

For $\int \frac{2}{x-1} dx \Rightarrow$ let $u = x-1$

$$\frac{du}{dx} = 1$$

$$\int \frac{2}{x-1} dx = \int \frac{2}{u} du \Rightarrow 2 \int \frac{1}{u} du$$

$$= 2 \ln u = 2 \ln(x-1)$$

Therefore $\int \frac{11-3x}{x^2+2x-3} = 2 \ln(x-1) - 5 \ln(x+3) + C$

2. $\int \frac{4x-16}{x^2-2x-3} dx$

$$x^2-2x-3 = (x-3)(x+1)$$

$$\frac{4x-16}{x^2-2x-3} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$\frac{4x-16}{x^2-2x-3} = \frac{A(x+1) + B(x-3)}{(x+3)(x+1)}$$

$$4x-16 = A(x+1) + B(x-3)$$

$$f(3) = 4(3)-16 = A(3+1)$$

$$12-16 = 4A$$

$$-4 = 4A$$

$$\therefore A = \underline{\underline{-1}}$$

$$f(-1) \Rightarrow 4(-1)-16 = B(-1-3)$$

$$-4-16 = -4B$$

Continuation of No. 2.

$$-20 = -4B$$

$$\therefore B = \underline{\underline{5}}$$

$$\int \frac{4x-16}{x^2-2x-3} dx = \int \frac{-1}{x-3} dx + \int \frac{5}{x+1} dx$$

$$\int \frac{-1}{x-3} dx \Rightarrow \text{let } u = x-3$$

$$\frac{du}{dx} = 1, \therefore dx = du$$

$$\int \frac{-1}{x-3} dx = \int \frac{-1}{u} du = -1 \ln u$$

$$= -\ln(x-3)$$

$$\int \frac{5}{x+1} \Rightarrow \text{let } u = x+1$$

$$du = dx$$

$$\int \frac{5}{x+1} dx = \int \frac{5}{u} du \Rightarrow 5 \int \frac{1}{u} du$$

$$5 \ln u = 5 \ln(x+1)$$

$$\int \frac{4x-16}{x^2-2x-3} dx = -\ln(x-3) + 5 \ln(x+1) + C$$

$$\int \frac{4x-16}{x^2-2x-3} \Rightarrow \underline{\underline{5 \ln(x+1) - \ln(x-3) + C}}$$

Question 3.

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$2x^2 - 9x - 35 = A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2)$$

$$2x^2 - 9x - 35 = (Ax^2 + Ax - 6A) + (Bx^2 + 4Bx + 3B) + (Cx^2 - Cx - 2C)$$

$$2x^2 - 9x - 35 = (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C)$$

$$A+B+C = 2 \quad \text{--- (1)}$$

$$A+4B-C = -9 \quad \text{--- (2)}$$

$$(-6A+3B-2C) = -35 \quad \text{--- (3)}$$

$$\text{From (1) } A = 2 - B - C \quad \text{--- (4)}$$

From equation (2), Substituting (4) into (2) we have

$$2 - B - C + 4B - C = -9$$

$$4B - B - C - C = -9 - 2$$

$$3B - 2C = -11 \quad \text{--- (5)}$$

From equation (3)

$$-6(2 - B - C) + 3B - 2C = -35$$

$$-12 + 6B + 6C + 3B - 2C = -35$$

$$6B + 3B + 6C - 2C = -35 + 12$$

$$9B + 4C = -23 \quad \text{--- (6)}$$

$$\text{Equation 5-6 : } 9 \times 3B - 2C = -11$$

$$3 \times 9B + 4C = -23$$

$$27B - 18C = -99$$

$$\underline{-27B + 12C = -69}$$

$$-30C = -30$$

Continuation of no. 3.

Also,

$$27B - 18(1) = -99$$

$$27B - 18 = -99$$

$$27B = -99 + 18$$

$$27B = -81$$

$$B = \underline{\underline{-3}}$$

Finally, $A + B + C = 2$

$$A - 3 + 1 = 2 \quad (\text{Substituting } C=1 \text{ and } B=-3)$$

$$A - 2 = 2$$

$$A = 2 + 2 = \underline{\underline{4}}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{(x+1)} dx + \int \frac{-3}{(x-2)} dx + \int \frac{1}{(x+3)} dx$$

For $\int \frac{4}{(x+1)} dx$; let $u = x+1$, $\frac{du}{dx} = 1$, $du = dx$

$$\int \frac{4}{u} du = 4 \ln u = 4 \ln(x+1)$$

For $\int \frac{-3}{(x-2)} dx = -3 \ln u = -3 \ln(x-2)$

For $\int \frac{1}{(x+3)} dx = \ln(x+3)$

$$\therefore \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \underline{\underline{4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3) + C}}$$