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NBBS

Assignment ^{$u = x^2 + 1$} $\frac{du}{dx}$

1) $\int \frac{11-3x}{(x^2+2x-3)} dx$

2) $\int \frac{4x-16}{(x^2-2x-3)} dx$

3) $\int \frac{(2x^2-9x-35)}{(x+1)(x-2)(x+3)}$

Solution

1) $\int \frac{11-3x}{(x^2+2x-3)} dx$

$$\frac{11-3x}{(x+3)(x-1)} = \frac{A}{(x+3)} + \frac{B}{(x-1)}$$

$$\frac{11-3x}{(x+3)(x-1)} = \frac{A(x-1) + B(x+3)}{(x+3)(x-1)}$$

$$11-3x = A(x-1) + B(x+3)$$

f(-3)

$$11-3(-3) = A(-3-1)$$

$$20 = -4A$$

$$A = -5$$

f(1) $\Rightarrow 11-3(1) = B(1+3)$

$$\Rightarrow 8 = 4B$$

$$B = 2$$

$$\int \frac{11-3x}{(x+3)(x-1)} dx = \int \frac{-5}{(x+3)} dx + \int \frac{2}{(x-1)} dx$$

let

$$\int \frac{11-3x}{(x+3)(x+1)} = \int \frac{-5}{x}$$

$$\int \frac{-5}{(x+3)}$$

$$\text{let } u = x+3$$

$$\frac{du}{dx} = 1$$

$$dx = \frac{du}{1}$$

$$\int \frac{-5}{u} \cdot \frac{du}{1}$$

$$-5 \int \frac{1}{u} du$$

$$= -5 \ln(u)$$

$$= -5 \ln(x+3)$$

$$+ \int \frac{2}{x-1} dx$$

$$\text{let } u = x-1$$

$$\frac{du}{dx} = 1$$

$$dx = \frac{du}{1}$$

$$= \int \frac{2}{u} \cdot du$$

$$= 2 \int \frac{1}{u} du$$

$$= 2 \ln(u)$$

$$= 2 \ln(x-1)$$

$$\therefore \int \frac{11-3x}{(x^2+2x-3)} = 2 \ln(x-1) - 5 \ln(x+3)$$

$$2) \int \frac{4x-16}{(x^2-2x-3)}$$

$$\frac{4x-16}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$\frac{4x-16}{(x-3)(x+1)} = \frac{A(x+1) + B(x-3)}{(x-3)(x+1)}$$

$$4x-16 = A(x+1) + B(x-3)$$

~~f(3)~~

$$f(3) \Rightarrow 4(3)-16 = A(3+1)$$

$$-4 = 4A$$

$$A = -1$$

$$\Rightarrow -20 = B - 4B$$

$$B = \frac{20}{-3}$$

$$B = 5$$

$$\frac{4x-16}{(x-3)(x+1)} = \int \frac{-1}{x-3} dx + \int \frac{5}{x+1} dx$$

$$\frac{-1}{(x-3)} \quad \text{let } U = (x-3)$$

$$= \frac{5}{(x+1)} \quad \text{let } U = x+1$$

$$= \frac{\delta U}{\delta x} = 1$$

$$= \frac{\delta U}{\delta x} = 1$$

$$= \delta x = \frac{\delta U}{1}$$

$$= \delta x = \frac{\delta U}{1}$$

$$\Rightarrow \int \frac{-1}{(x-3)} \cdot \frac{\delta U}{1}$$

$$= \int \frac{5}{U} \cdot \frac{\delta U}{1}$$

$$= -1 \int \frac{1}{U} \cdot \frac{\delta U}{1}$$

$$= 5 \int \frac{1}{U} \delta U$$

$$= -1 \ln U$$

$$= 5 \ln U$$

$$\therefore \int \frac{4x-16}{x^2-2x-3} = 5 \ln U - \ln U$$

$$\int \frac{4x-16}{x^2-2x-3} = 5 \ln(x+1) - \ln(x-3)$$

$$3) \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

$$f(-1) \Rightarrow 2x^2 - 9x - 35 = A(x-2)(x+3)$$

$$\Rightarrow 2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3)$$

$$\Rightarrow -24 = -6A$$

$$A = \frac{-24}{-6}$$

$$A = 4$$

$$f(2) \Rightarrow 2(2)^2 - 9(2) - 35 = B(2+1)(2+3)$$

$$\Rightarrow 8 - 18 - 35 = 15B$$

$$\Rightarrow -45 = 15B$$

$$B = -3$$

$$-3) \Rightarrow 2(-3)^2 - 9(-3) - 35 = C(-3+1)(-3-2)$$

$$\Rightarrow 18 + 27 - 35 = 10C$$

$$\Rightarrow 10 = 10C$$

$$C = \frac{10}{10}$$

$$C = 1$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{(x+1)} dx + \int \frac{-3}{(x-2)} dx + \int \frac{1}{(x+3)} dx$$

$$\Rightarrow \int \frac{4}{(x+1)} dx + \int \frac{-3}{(x-2)} dx + \int \frac{1}{(x+3)} dx$$

let $u = x+1$ let $v = x-2$ let $w = x+3$
 $\frac{du}{dx} = 1$ $\frac{dv}{dx} = 1$ $\frac{dw}{dx} = 1$
 $dx = \frac{du}{1}$ $dx = \frac{dv}{1}$ $dx = \frac{dw}{1}$

$$\int \frac{4}{u} \frac{du}{1} = \int \frac{-3}{v} \frac{dv}{1} = \int \frac{1}{w} \frac{dw}{1}$$

$$+ \int \frac{1}{u} du = -3 \int \frac{1}{v} dv = \int \frac{1}{w} dw$$

$$+ \ln u = -3 \ln v = -\ln w$$

$$\ln(x+1) + \ln(x+3) - 3 \ln(x-2)$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x-3)} dx = 4 \ln(x+1) + \ln(x+3) - 3 \ln(x-2)$$

or

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x-3)} dx = 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)$$