NAME: AWALA DIVINE PAUL
MATRIC N0: 19/ENG05/016
DEPARTMENT: MECHATRONICS ENGINEERING.
COURSE CODE: MAT 102

COVID-19 HOLIDAY ASSIGNMENT

1. If A = $5i - 7j - 6k$, B = $j + 4k$, C = $9i - 4j + k$. Find $-8(A + B) . (C - A)$

   SOLUTION
   $-8(A + B) . (C - A)$
   $(A + B) = [(5i - 7j - 6k) + (j + 4k)]$
   $= [5i - 6j - 2k]$
   $-8(A + B) = -8[5i - 6j - 2k]$
   $= [-40i + 48j + 16k]$
   $(C - A) = [(9i - 4j + k) - (5i - 7j - 6k)]$
   $= [4i + 3j + 7k]$
   $-8(A + B) . (C - A) = [-40i + 48j + 16k] . [4i + 3j + 7k]$
   $= -160 + 144 + 112$
   $= 96$
   $\therefore -8(A + B) . (C - A) = 96$

2. Find a unit vector tangent to the space curve $x = -3t$, $y = t^2$, $z = 4t^3$ at the point where $t = 1$

   SOLUTION
   $\vec{r} = xi + yj + zk$
   $\vec{r} = -3ti + t^2 j + 4t^3 k$
   $T = \frac{d\vec{r}/dt}{|dr/dt|}$
   $\frac{d\vec{r}}{dt} = -3i + 2tj + 12t^2 k$ where $t = 1$
   $\frac{d\vec{r}}{d_{t=1}} = -3i + 2j + 12k$
   $\left|\frac{dr}{dt}\right|_{t=1} = \sqrt{(-3^2) + 2^2 + 12^2} = \sqrt{9 + 4 + 144} = \sqrt{157} = 12.53$
   Hence, $T = \frac{d\vec{r}/dt}{|dr/dt|} = \frac{-3i + 2tj + 12t^2 k}{12.53}$
   $\therefore T = \frac{-3i + 2tj + 12t^2 k}{12.53}$

3. A particle moves along a curve, $x = 8t^2$, $y = t^2 - 4t$, $z = t + 1$, where *t* is time. Find its acceleration.

SOLUTION

$\vec{r} = \text{x}i + \text{y}j + \text{z}k$

$\vec{r} = (8t^2)i + (t^2 - 4t)j + (t + 1)k$

$\frac{d\vec{r}}{dt}$ = velocity = *v*

$\frac{d\vec{r}}{dt}$ = (16t)i + (2t – 4)j + k

$\frac{d^2\vec{r}}{dt}$ = (16)i + (2)j + k

∴ $\frac{d^2\vec{r}}{dt^2}$ = Acceleration = 16i + 2j

4. If A = i + 2j – 4k, B = 2i – 3j + k, C = 4j – 3k. Find (A × B) × C.

SOLUTION

(A × B) × C

(A × B) = $\begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 2 & -3 & 1 \end{vmatrix}$

(A × B) = [(2 × 1) – 12]i – [(1 × 1) – (– 8)]j + [(– 3 × 1) – 4]k

= [2 – 12]i + [1 + 8]j + [– 3 – 4]k

= –10i + 9j – 7k

(A × B) × C = $\begin{vmatrix} i & j & k \\ -10 & 9 & -7 \\ 0 & 4 & -3 \end{vmatrix}$

(A × B) × C = [(9 × – 3) – (– 28)]i – [(– 10 × – 3) – 0]j + [(– 10 × 4) – 0]k

= [– 27 + 28]i – [30 – 0]j + [– 40 – 0]k

= i – 30j – 40k

∴ (A × B) × C = i – 30j – 40k

5. Given R = $(4\sin 3t)i + (4e^{3t})j + (7t^3)k$. Find the integral of R with respect to *t* from 0 to 1

SOLUTION

$\int_0^1 Rdq = \int_0^1 (4\sin 3t)i + (4e^{3t})j + (7t^3)k]dt$

$= \int (4\sin 3t)i\ dt + \int (4e^{3t})j\ dt + \int (7t^3)k\ dt$

$= (4 \times \frac{-1}{3}\cos 3t)i + (4 \times \frac{1}{3}e^{3t})j + (\frac{7t^4}{4})k$

$\int_0^1 Rdq = \int_0^1 (4\sin 3t)i + (4e^{3t})j + (7t^3)k]dt = (\frac{-4}{3}\cos 3t)i + (\frac{4}{3}e^{3t})j + (\frac{7t^4}{4})k$

$= (\frac{-4}{3}\cos 3)i + (\frac{4}{3}e^{3})j + (\frac{7}{4})k - [(\frac{-4}{3}\cos 0)i + (\frac{4}{3}e^{0})j + (\frac{0}{4})k]$

$= (\frac{-4}{3} \times 0.9986)i + (\frac{4}{3} \times 20.0855)j + (\frac{7}{4})k - [(\frac{-4}{3} \times 1)i + (\frac{4}{3} \times 1)j]$

= – 1.3315i + 26.7807j + 1.75k + 1.3333i – 1.3333j

∴ $\int_0^1 Rdq$ = 0.0018i + 25.4474j + 1.75k