

## MAT 104 MBBS Assignment

$$1. \int \frac{11-3x}{x^2+2x-3} dx$$

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x^2+3x)(x-3)} = \frac{11-3x}{x(x+3)-1(x+3)} = \frac{11-3x}{(x-1)(x+3)}$$

$$\therefore \int \frac{11-3x}{(x-1)(x+3)} = \int \frac{A}{(x-1)} + \int \frac{B}{(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

By comparing numerators

$$11-3x \equiv A(x+3) + B(x-1)$$

$$11-3x \equiv A(x+3) + B(x-1)$$

$$11-3x \equiv Ax + 3A + Bx - B$$

$$11-3x \equiv (A+B)x + (3A-B)$$

By comparison

$$A+B = -3 \quad \text{--- (1)}$$

$$+ (3A-B = 11 \quad \text{--- (2)})$$

$$4A = 8$$

$$A = \frac{8}{4} = 2$$

Put  $A=2$  into eqn (1)

$$2+B = -3$$

$$B = -3-2 = -5$$

$$\therefore \int \frac{11-3x}{(x-1)(x+3)} dx = \int \frac{2}{(x-1)} dx + \int \frac{-5}{(x+3)} dx$$

$$= 2 \int \frac{1}{(x-1)} dx - 5 \int \frac{1}{(x+3)} dx$$

$$\int \frac{11-3x}{(x-1)(x+3)} dx = 2 \ln(x-1) - 5 \ln(x+3) + C$$

$$2 \int \frac{4x-16}{x^2-2x-3} dx$$

$$= \int \frac{4x-16}{x^2-3x+x-3} dx = \int \frac{4x-16}{x(x-3)+1(x-3)}$$

$$= \int \frac{4x-16}{(x+1)(x-3)}$$

$$\int \frac{4x-16}{(x+1)(x-3)} dx = \int \frac{A}{(x+1)} + \frac{B}{(x-3)} dx$$

$$\int \frac{4x-16}{(x+1)(x-3)} dx \equiv \frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$$

By equating numerators

$$4x-16 \equiv A(x-3) + B(x+1)$$

$$4x-16 \equiv Ax-3A+Bx+B$$

$$4x-16 \equiv Ax+Bx-3A+B$$

$$4x-16 \equiv (A+B)x - (3A+B)$$

By comparison

$$A+B = 4 \quad \text{--- ①}$$

$$-(-3A+B = -16) \quad \text{--- ②}$$

$$4A = 20$$

$$A = 20/4 = 5 //$$

Put  $A=5$  into eqn ①

$$5+B=4$$

$$B=4-5=-1$$

$$\therefore \int \frac{4x-16}{(x+1)(x-3)} dx = \int \frac{5}{(x+1)} dx + \int \frac{-1}{(x-3)} dx$$

$$\int \frac{4x-16}{(x+1)(x-3)} dx = 5 \ln(x+1) - \ln(x-3) + c$$

$$3 \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3} dx$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)} dx$$

By equating numerators

$$2x^2 - 9x - 35 \equiv A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

$$2x^2 - 9x - 35 \equiv A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2)$$

$$2x^2 - 9x - 35 \equiv Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$2x^2 - 9x - 35 \equiv Ax^2 + Bx^2 + Cx^2 + Ax + 4Bx - Cx - 6A + 3B - 2C$$

$$2x^2 - 9x - 35 = (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C)$$

$$\therefore A + B + C = 2 \quad \text{--- (1)}$$

$$A + 4B - C = -9 \quad \text{--- (2)}$$

$$-6A + 3B - 2C = -35 \quad \text{--- (3)}$$

From eqn (1)  $A + B + C = 2$

$$C = 2 - A - B \quad \text{--- (4)}$$

Put (4) in eqn (2)  $A + 4B - [2 - A - B] = -9$

$$A + 4B - 2 + A + B = -9$$

$$2A + 5B = -9 + 2$$

$$2A + 5B = -7 \quad \text{--- (5)}$$

Put (4) in eqn (3)  $-6A + 3B - 2[2 - A - B] = -35$

$$-6A + 3B - 4 + 2A + 2B = -35$$

$$-4A + 5B = -35 + 4$$

$$-4A + 5B = -31 \quad \text{--- (6)}$$

$$2A + 5B = -7 \quad \text{--- (5)}$$

$$- \left[ -4A + 5B = -31 \quad \text{--- (6)} \right]$$

$$6A = 24$$

$$A = \frac{24}{6} = 4$$

Put  $A = 4$  into eqn (5)

$$2(4) + 5B = -7$$

$$8 + 5B = -7$$

$$5B = -15$$

$$B = -15/5 = -3$$

Put  $A = 4$ ,  $B = -3$  into eqn (4)

$$C = 2 - 4 - (-3)$$

$$C = 2 - 4 + 3$$

$$C = 1$$

$$\therefore \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} dx + \int \frac{-3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)$$