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MTH 04th

$$\int \frac{11-3x}{x^2-2x-3} dx = \frac{11-3x}{(x-1)(x+3)}$$

$$\frac{A(x+3) + B(x-1)}{(x-1)(x+3)} = \frac{11-3x}{(x-1)(x+3)}$$

Cancel both sides by $(x-1)(x+3)$

$$A(x+3) + B(x-1) = 11-3x$$
$$Ax + 3A + Bx - B = 11 - 3x$$
$$x(A+B) + 3A - B = 11 - 3x$$

$$\therefore A+B = -3$$

$$3A - B = 11$$

To find B

$$A = -3 - B$$

$$3(-3 - B) - B = 11$$

$$-9 - 3B - B = 11$$

$$-4B = 20$$

$$B = -5$$

To find A

$$B = -3 - A$$

$$3A - (-3 - A) = 11$$

$$3A + 3 + A = 11$$

$$4A = 8$$

$$A = 2$$

$$\therefore \int \frac{11-3x}{x^2-2x-3} = \int \frac{A}{x-1} - \int \frac{B}{x+3} = \int \frac{11-3x}{(x-1)(x+3)} = \int \frac{2}{x-1}$$
$$= \int \frac{2}{x-1} = 2 \ln|x-1| - 5 \ln|x+3| + C$$

$$2) \int \frac{4x-16}{x^2-2x-3} dx = \int \frac{4x-16}{(x-3)(x+1)}$$

$$\frac{4x-16}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$\text{let } x = -1$$

$$4(-1)-16 = A(-1-1) + B(-1-1)$$

$$0 + B(4) = 6 - 16$$

$$4B = -10$$

$$B = -\frac{5}{4}$$

$$\text{let } x = 3$$

$$4(3)-16 = A(3+1) + B(3-3)$$

$$4A + 0 = 6$$

$$A = \frac{3}{2}$$

$$\int \frac{4x-16}{x^2-2x-3} dx = \int \frac{\frac{3}{2}}{x-3} + \int \frac{-\frac{5}{4}}{x+1}$$

$$\int \frac{4x-16}{x^2-2x-3} = \int \frac{1.5}{x-3} + \int \frac{-1.25}{x+1}$$

$$\int \frac{4x-16}{(x^2-2x-3)} = 1.5 \ln|x-3| - 1.25 \ln|x+1|$$

$$3) \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$2x^2 - 9x - 35 = A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2)$$

$$2x^2 - 9x - 35 = Ax^2 + Ax + 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$2x^2 - 9x - 35 = x^2(A+B+C) + x(A+4B-C) - 6A + 3B - 2C$$

Hence

$$A + B + C = 2 \quad \text{--- (i)}$$

$$A + 4B - C = -9 \quad \text{--- (ii)}$$

$$-6A + 3B - 2C = -35 \quad \text{--- (iii)}$$

$$A = 2 - B - C \quad \text{--- (iv)}$$

$$2 - B - C + 4B - C = 9$$

$$-2C + 3B = 7 \quad \text{--- (v)}$$

$$\begin{aligned} \text{Sub (1) into (2)} \\ -6(2-B-C) + 3B - 2C = -25 \\ -12 + 6B + 6C + 3B - 2C = -25 \\ 4C + 9B = -23 \quad (3) \end{aligned}$$

$$\begin{aligned} -2 + 9B &= -11 \\ 4C + 9B &= -23 \end{aligned}$$

$$\begin{aligned} -6C + 9B &= 33 \\ 4C + 9B &= -23 \\ -10C &= -10 \\ C &= 1 \end{aligned}$$

$$\begin{aligned} 4C + 9B &= -23 \\ 4(1) + 9B &= -23 \\ 9B &= -27 \\ B &= -3 \end{aligned}$$

$$\begin{aligned} \text{For A} \\ A &= 2 - B - C \\ A &= 2 + 3 - 1 \\ A &= 4 \end{aligned}$$

$$\therefore \int \frac{2x^2 - 9x - 25}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} - \int \frac{3}{x-2} + \int \frac{1}{x+3}$$

$$= 4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3| + C$$