ESHIET MFONGBONG UDO - MAT104

19/MHS01/161

3) $\int \frac{11-3x}{x^2+2x-3} dx$

Solution

$$\int \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

$$11-3x = A(x+3)+B(x-1)$$

$$f(1) = 11-3(1) = A(1+3)$$

$$\therefore \frac{8}{4} = \frac{4A}{4}$$

$$A = 2$$

$$f(-3) \therefore 11-3(-3) = B(-3-1)$$

$$B = 5$$

we can now write.

$$\frac{11-3x}{(x-1)(x+3)} = \frac{2}{(x-1)} + \frac{5}{(x+3)}$$

$$= \int \frac{2dx}{x-1} + \int \frac{5\,dx}{(x+3)} = \int \frac{11-3x}{x^2+2x-3}$$

Let $u = x-1$ Let $u = x+3$

$du = 1dx$ $du = dx$

$dx = du$ $dx = du$

$$\Rightarrow 2\int \frac{du}{u} + 5\int \frac{du}{u}$$

$$= 2\ln u + 5\ln u$$

$$\Rightarrow 2\ln(x-1) + 5\ln(x+3) + C$$

2) $\int \frac{4x-16}{x^2-2x-3)} dx$

Solution

$$\int \frac{4x-16}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$\frac{4x-16}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$\frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$$

$$\frac{4x-16}{(x+1)(x-3)} = \frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$$

$$4x-16 = A(x-3) + B(x+1)$$

$f(-1)$ $\therefore$ $4(-1) - 16 = A(-1-3)$

$$-20 = -4A$$

$$A = 5$$

$f(3)$ $\therefore$ $4(3) - 16 = B(3+1)$

$$= 12 - 16 = 4B$$

$$B = -1$$

$$\frac{4x-16}{(x+1)(x-3)} = \frac{5}{(x+1)} - \frac{1}{(x-3)}$$

$$= \int \frac{5dx}{x+1} - \int \frac{1}{x-3} = \int \frac{4x-16}{x^2-2x-3}$$

let $u = x+1$ let $u = x-3$

$du = dx$ $du = dx$

$dx = du$ $dx = du$

$$\Rightarrow 5\int \frac{du}{u} - \int \frac{du}{u}$$

$$\Rightarrow 5\ln u - \ln u$$

$$\Rightarrow 5\ln(x+1) - \ln(x-3) + c$$

3 $$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx$$

Solution.

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{A(x-2)(x+3) + B(x+1)(x+3 + C(x+1)(x+}{(x+1)(x-2)(x+3)}$$

$$2x^2-9x-35 = A(x^2+x-6) + B(x^2+4x+3) + C(x^2-x-2)$$

$$2x^2-9x-35 = Ax^2+Ax-6A + Bx^2+4Bx+3B + Cx^2-Cx-2C$$

$$2x^2-9x-35 = Ax^2+Bx^2+Cx^2+Ax+4Bx-Cx-6A+3B-2C$$

$2x^2 - 9x - 35 = (A + B + C)x^2 + (A + 4B - C)x + (-6A + 3B - 2C)$

$A + B + C = 2$ -- (1)

$A + 4B - C = -9$ -- (2)

$-6A + 3B - 2C = -35$ -- (3)

From (1) $A = 2 - B - C$ -- (4) & put in (2) & (3)

Eq (2) $2 - B - C + 4B - C = -9 \therefore 2 + 3B - 2C = -9$

Eq (3) $-6(2 - B - C) + 3B - 2C = -35 \therefore -12 + 6B + 6C + 3B - 2C = -35$

$\therefore -12 + 9B + 4C = -35$

$9B + 4C = -35 + 12$

$+4C = -23$

$9\left(\frac{-11 + 2C}{3}\right) = -23$

Also $3B - 2C = -11$

$B = \frac{-11 + 2C}{3}$

$\therefore \frac{-99 + 18C}{3} + 4C = -23$

$-33 + 6C + 4C = -23$

$10C = 33 - 23$

$C = 1$

$3B - 2(1) = -11$

$3B - 2 = -11$

$3B = -11 + 2$

$B = \frac{-9}{3}$

$B = -3$

Finally $A + B + C = 2$

$A + (-3) + (1) = 2$

$A + (-2) = 2$

$A = 2 + 2 = 4$

$$\int \frac{2x^2-x-35}{(x+1)(x-2)(x+3)} = \int \frac{4\,dx}{x+1} + \int \frac{-3\,dx}{x-2} + \int \frac{1}{x+3}\,dx$$

$$= 4\ln u - 3\ln u + \ln u + C$$

$$= 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + C$$