

$$1. \int \frac{11-3x}{x^2+2x-3} dx$$

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

Comparing, $11-3x = A(x+3) + B(x-1)$

At $x=1$; $f(1) \Rightarrow 11-3(1) = A(1+3) + B(1-1)$

$$8 = 4A$$

$$A = 2$$

At $x=-3$; $f(-3) \Rightarrow 11-3(-3) = A(-3+3) + B(-3-1)$

$$20 = -4B$$

$$B = -5$$

$$\frac{11-3x}{x^2+2x-3} = \frac{2}{x-1} - \frac{5}{x+3}$$

$$\therefore \int \frac{11-3x}{x^2+2x-3} dx = \int \frac{2}{x-1} dx - \int \frac{5}{x+3} dx$$

$$= 2 \int \frac{dx}{x-1} - 5 \int \frac{dx}{x+3}$$

$$\int \frac{11-3x}{x^2+2x-3} dx = 2 \ln(x-1) - 5 \ln(x+3) + k$$

Where k is the constant of integration

$$2 \int \frac{4x-16}{x^2-2x-3} dx$$

$$\frac{4x-16}{x^2-2x-3} = \frac{4x-16}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$\frac{4x-16}{(x+1)(x-3)} = \frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$$

Comparing, $4x-16 = A(x-3) + B(x+1)$

At $x = -1$; $f(-1) \Rightarrow 4(-1) - 16 = A(-1-3) + B(-1+1)$
 $-20 = -4A$

$$A = 5$$

At $x = 3$; $f(3) \Rightarrow 4(3) - 16 = A(3-3) + B(3+1)$
 $-4 = 4B$

$$B = -1$$

$$\frac{4x-16}{x^2-2x-3} = \frac{5}{x+1} + \left(\frac{-1}{x-3}\right) = \frac{5}{x+1} - \frac{1}{x-3}$$

$$\int \frac{4x-16}{x^2-2x-3} dx = \int \frac{5}{x+1} dx - \int \frac{dx}{x-3}$$

$$\therefore \int \frac{4x-16}{x^2-2x-3} dx = 5 \ln(x+1) - \ln(x-3) + K$$

Where K is the constant of integration

$$3 \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

Comparing;

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

At $x=2$

$$f(2) \Rightarrow 2(2^2) - 9(2) - 35 = A(2-2)(2+3) + B(2+1)(2+3) + C(2+1)(2-2)$$

$$-15 = 15B$$

$$B = -3$$

At $x=-1$;

$$f(-1) \Rightarrow 2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3) + B(-1+1)(-1+3) + C(-1+1)(-1-2)$$

$$-24 = -6A$$

$$A = 4$$

At $x=1$

$$f(1) \Rightarrow 2(1^2) - 9(1) - 35 = A(1-2)(1+3) + B(1+1)(1+3) + C(1+1)(1-2)$$

$$-42 = A(-4) + B(8) + C(-2)$$

$$-42 = 4(-4) + -3(8) + C(-2)$$

$$-42 = -16 - 24 - 2C$$

$$2C = -16 - 24 + 42$$

$$2C = 2 ; C = 1$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} + \frac{-3}{x-2} + \frac{1}{x+3}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} dx - \int \frac{3}{x-2} dx + \int \frac{dx}{x+3}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3) + k$$

where k is the constant of integration