

MAT Assignment

$$\int \frac{11-3x}{(x^2+2x-3)} dx$$

Solution

x^2+2x-3	$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)}$
$x^2+3x-x-3$	$= \frac{A}{x-1} + \frac{B}{x+3} \Rightarrow \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$
$x(x+3) - 1(x+3)$	
$(x-1)(x+3)$	

$$A(x+3) + B(x-1) = 11-3x$$

$$Ax + 3A + Bx - B = 11-3x$$

$$(A+B)x + (3A-B) = 11-3x$$

(Comparing) $A+B = -3 \dots \textcircled{1}$

$3A-B = 11 \dots \textcircled{2}$

$4A = 8 \Rightarrow A=2$

$2+B = -3$ (Put in equation 1)

$2+B = -3$

$B = -3-2 \Rightarrow B = -5$

We can now write

$$\int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx = \int \frac{11-3x}{x^2+2x-3}$$

let $u = x-1$

$u = x+3$

$du = dx$

$du = dx$

(-5)

We can now write

$$\int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx = \int \frac{11-3x}{x^2+2x-3}$$

$$\text{let } u = x-1$$

$$u = x+3$$

$$du = dx$$

$$du = dx$$

$$\Rightarrow \int \frac{2 du}{u}$$

$$\Rightarrow \int \frac{-5 du}{x+3u}$$

$$= 2 \ln u$$

$$\Rightarrow -5 \ln u$$

$$\Rightarrow 2 \ln(x-1) - 5 \ln(x+3) + C$$

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$$\int \frac{4x-16}{x^2-2x-3} dx$$

Solution

$$\int \frac{4x-16}{x^2-2x-3} dx \Rightarrow \int \frac{4x-16}{(x+1)(x-3)}$$

$$\frac{x^2-2x-3}{x^2-3x+x-3} = \frac{A}{x+1} + \frac{B}{x-3} \Rightarrow \frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$$

$$\frac{x(x-3) + 1(x-3)}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$A(x-3) + B(x+1) = 4x-16$$

$$Ax - 3A + Bx + B = 4x - 16$$

$$(B+A)x + (B-3A) = 4x - 16$$

(Comparing sides) $B+A = 4$... (1)

$$B-3A = -16$$
 ... (2)

$$4A = 20 \Rightarrow A = 5$$

$$4, 4, \text{ (Put in equation (1))}$$

$$B+5=4$$

$$B = 4-5 \Rightarrow B = -1$$

We can now write

$$\int \frac{5}{x+1} dx + \int \frac{-1}{x-3} dx = \int \frac{4x-16}{x^2-2x-3} dx$$

$$\text{let } u = x+1$$

$$u = x-3$$

$$du = dx$$

$$du = dx$$

$$\Rightarrow \int \frac{5 du}{u}$$

$$\Rightarrow \int \frac{-du}{u}$$

$$\Rightarrow 5 \ln u$$

$$\Rightarrow -\ln u$$

$$\Rightarrow 5 \ln(x+1) - \ln(x-3) + C$$

$$\int \frac{(2x^2 - 9x - 35) dx}{(x+1)(x-2)(x+3)}$$

Solution

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$2x^2 - 9x - 35 = A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 + x - 2)$$

$$2x^2 - 9x - 35 = Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$2x^2 - 9x - 35 = Ax^2 + Bx^2 + Cx^2 + Ax + 4Bx - Cx - 6A + 3B - 2C$$

$$2x^2 - 9x - 35 = (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C)$$

(Comparing) $A+B+C = 2 \dots \textcircled{1}$

$$A+4B-C = -9 \dots \textcircled{2}$$

$$-6A+3B-2C = -35 \dots \textcircled{3}$$

$$A+B+C = 2$$

$$A+4B-C = -9 \quad \textcircled{4}$$

$$-3B+2C = 11, \therefore 3B-2C = -11 \text{ (Rt in equation } \textcircled{3})$$

$$\therefore -6A(-11) = -35$$

$$6A = -35 + 11 = -24, \therefore A = 4$$

$$\therefore 4+B+C=2 \Rightarrow B+C=2-4 \Rightarrow B+C=-2$$

$$4+4B-C=-9 \Rightarrow 4B-C=-9-4 \Rightarrow 4B-C=-13$$

$$B+C=-2 \dots (5)$$

$$+ 4B-C=-13 \dots (6)$$

$$\frac{15B}{5} = \frac{-153}{5}, \therefore B = -3 \text{ (put in equation 5)}$$

Finally, $-3 + C = -2$

$$C = -2 + 3$$

$$C = 1$$

$$\therefore A=4, B=-3, C=1 \therefore A=4, B=-3, C=1$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} dx + \int \frac{-3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$\begin{array}{lll} \text{let } u = x+1 & u = x-2 & u = x+3 \\ du = dx & du = dx & du = dx \end{array}$$

$$\Rightarrow 4 \int \frac{du}{u} - 3 \int \frac{du}{u} + \int \frac{du}{u}$$

$$\Rightarrow 4 \ln u - 3 \ln u + \ln u$$

$$\Rightarrow 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)$$