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DEPARTMENT: MECHATRONICS ENGINEERING.

COURSE CODE: MAT 102

COVID-19 HOLIDAY ASSIGNMENT

1. A particle moves along a curve, $x = t^2$, $y = -5t^2 + t$, $z = t + 7$, where t is time. Find its acceleration

SOLUTION

$$\vec{r} = xi + yj + zk$$

$$\vec{r} = (t^2)i + (-5t^2 + t)j + (t + 7)k$$

$$\frac{d\vec{r}}{dt} = \text{velocity} = v$$

$$\frac{d\vec{r}}{dt} = (2t)i + (-10t + 1)j + k$$

$$\frac{d^2\vec{r}}{dt^2} = (2)i + (-10)j$$

$$\therefore \frac{d^2\vec{r}}{dt^2} = \text{Acceleration} = 2i - 10j$$

2. If $P = i - 9j - 4k$, $Q = 8i - 3j + 6k$, $R = i - 4j - 3k$, Find $(P \times Q) \cdot (R \times P)$.

SOLUTION

$$(P \times Q) \cdot (R \times P)$$

$$(P \times Q) = \begin{vmatrix} i & j & k \\ 1 & -9 & -4 \\ 8 & -3 & 6 \end{vmatrix}$$

$$\begin{aligned} (P \times Q) &= [(-9 \times 6) - 12]i - [(6 \times 1) - (-32)]j + [(-3 \times 1) - (-72)]k \\ &= [-54 - 12]i + [6 + 32]j + [-3 + 72]k \\ &= -66i + 38j + 69k \end{aligned}$$

$$(R \times P) = \begin{vmatrix} i & j & k \\ 1 & -4 & -3 \\ 1 & -9 & -4 \end{vmatrix}$$

$$\begin{aligned} (R \times P) &= [(-4 \times -4) - 27]i - [(-4 \times 1) - (-3)]j + [(-9 \times 1) - (-4)]k \\ &= [16 - 27]i - [-4 + 3]j + [-9 + 4]k \\ &= -11i + j - 5k \end{aligned}$$

$$\begin{aligned} (P \times Q) \cdot (R \times P) &= [-66i + 38j + 69k] \cdot [-11i + j - 5k] \\ &= [726 + 38 - 345] \\ &= 419 \end{aligned}$$

$$\therefore (P \times Q) \cdot (R \times P) = 419$$

3. Given $F = (5 \cos 7t)i - (2e^{3t})j - (4t^3)k$, find the integral of F with respect to t .

SOLUTION

$$\int F dt = \int [(5 \cos 7t)i - (2e^{3t})j - (4t^3)k] dt$$

$$= \int (5 \cos 7t)i dt - \int (2e^{3t})j dt - \int (4t^3)k dt$$

$$= (5 \times \frac{1}{7} \sin 7t)i - (2 \times \frac{1}{3} e^{3t})j - (\frac{4t^4}{4})k$$

$$\int F dt = \int (5 \cos 7t)i dt - \int (2e^{3t})j dt - \int (4t^3)k dt = (\frac{5}{7} \sin 7t)i - (\frac{2}{3} e^{3t})j - (t^4)k$$

$$\therefore \int F dt = (\frac{5}{7} \sin 7t)i - (\frac{2}{3} e^{3t})j - (t^4)k + C$$