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MBBS
MAT 104

$$1) \int \frac{11-3x}{x^2+2x-3}$$

Solution

$$\begin{aligned}x^2+2x-3 &= x^2+3x-x-3 \\ &= x(x+3) - 1(x+3) \\ &= (x-1)(x+3)\end{aligned}$$

$$\Rightarrow \int \frac{11-3x}{x^2+2x-3} = \int \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$= \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

$$11-3x = A(x+3) + B(x-1)$$

$$11-3x = Ax + 3A + Bx - B$$

$$11-3x = Ax + Bx + 3A - B$$

$$11-3x = (A+B)x + (3A-B)$$

Comparing

$$A+B = -3 \quad \text{--- } i$$

$$3A-B = 11 \quad \text{--- } ii$$

$$\begin{array}{r} 3A+3B = -9 \quad \text{--- } iii \\ - \end{array}$$

$$3A - B = 11 \quad \text{--- } ii$$

$$4B = -20$$

$$B = -20/4 \quad \therefore B = -5$$

Sub 5 for B in eqn i

$$A+B = -3$$

$$A-5 = -3$$

$$A = -3+5$$

$$A = 2$$

$$\therefore \int \frac{A}{x-1} + \int \frac{B}{x+3} = \int \frac{11-3x}{x^2+2x-3} dx$$

$$= \int \frac{2 dx}{x-1} + \int \frac{-5 dx}{x+3} = \int \frac{11-3x}{x^2+2x-3} dx$$

$$\begin{array}{l} \text{Let } u=x-1 \quad u=x+3 \\ du=dx \quad du=dx \\ = \int \frac{2 du}{u} = \int \frac{-5 du}{u} \\ = 2 \ln u \quad -5 \ln u \end{array}$$

$$\therefore \int \frac{11-3x}{x^2+2x-3} = 2 \ln u + (-5 \ln u) = 2 \ln u - 5 \ln u$$

$$= 2 \ln(x-1) - 5 \ln(x+3)$$

$$2 \rightarrow \int \frac{4x-16}{x^2-2x-3}$$

Solution

$$x^2-2x-3 = x^2-3x+x-3$$

$$= x(x-3) + 1(x-3)$$

$$= (x+1)(x-3)$$

$$\Rightarrow \int \frac{4x-16}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$\frac{A(x+1)(x-3) + B(x+1)(x-3)}{(x+1)(x-3)}$$

$$4x-16 = A(x-3) + B(x+1)$$

$$4x-16 = Ax-3A+Bx+B$$

$$4x-16 = Ax+Bx-3A+B$$

$$4x-16 = (A+B)x + (-3A+B)$$

Comparing

$$A+B=4$$

$$-3A+B=-16$$

$$4A=20$$

$$A=5$$

Sub 5 for A in eqn 1

$$A+B=4$$

$$5+B=4$$

$$B=4-5$$

$$B=-1$$

$$\int \frac{A}{x+1} + \int \frac{B}{x-3} = \int \frac{4x-16}{x^2-2x-3}$$

$$\int \frac{5dx}{x+1} + \int \frac{-dx}{x-3}$$

$$\text{Let } u=x+1 \quad u=x-3$$

$$du=dx$$

$$du=dx$$

$$\int \frac{5du}{u}$$

$$\int \frac{du}{u}$$

$$= 5 \ln u$$

$$= -\ln u$$

$$2. \int \frac{4x-16}{x^2-2x-3} = 5 \ln(x+1) - \ln(x-3)$$

$$3. \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$$

$$(x+1)(x-2)(x+3)$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$= \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$= A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

$$= A(x^2+x-6) + B(x^2+4x+3) + C(x^2-x-2)$$

$$= Ax^2 - Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$= Ax^2 + Bx^2 + Cx^2 - Ax + 4Bx - Cx - 6A + 3B - 2C$$

$$2x^2 - 9x - 35 = (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C)$$

Comparing \Rightarrow

$$A+B+C = 2 \quad \text{--- (i)}$$

$$A+4B-C = -9 \quad \text{--- (ii)}$$

$$-6A+3B-2C = -35 \quad \text{--- (iii)}$$

$$A+B+C = 2$$

$$A+4B-C = -9$$

$$-3B+2C = 11 \quad \therefore 3B-2C = -11 \quad \text{--- (iv)}$$

(Put in eqn 3)

$$-6A+3B-2C = -35$$

$$-6A-11 = -35$$

$$-6A = -35+11$$

$$-6A = -24$$

$$A=4$$

$$4+B+C=2 \quad \therefore \quad B+C=-2$$

$$4+4B-C=-9 \quad 4B-C=-13$$

$$B+C=-2 \quad \text{--- (5)}$$

$$4B-C=-13 \quad \text{--- (6)}$$

$$5B=-15$$

$$B=-3$$

Sub -3 for B in eqn (5)

$$B+C=-2$$

$$-3+C=-2$$

$$C=-2+3$$

$$C=1$$

$$A=4, \quad B=-3, \quad C=1$$

$$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx = \int \frac{4dx}{x+1} + \int \frac{-3dx}{x-2} + \int \frac{dx}{x+3}$$

$$\text{Let } u=x+1 \quad u=x-2 \quad u=x+3$$

$$du=dx$$

$$= \int \frac{4dy}{u} = \int \frac{-3dy}{u} = \int \frac{dy}{u}$$

$$= 4 \ln u = -3 \ln u = \ln u$$

$$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx = 4 \ln u - 3 \ln u + \ln u$$
$$= \underline{4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)}$$