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MATRIC NUMBER: 19/MHS01/253

COURSE CODE: MAT 104

DEPARTMENT: MEDICINE AND SURGERY

1 $\frac{11+3x}{x^2+2x-3}$

Soln

$\int \frac{11-3x}{x^2+2x-3}\,dx$

From the denominator,

$x^2+2x-3=0$

$x^2+3x-x-3=0$

$x(x+3)-1(x+3)=0$

$(x-1)(x+3)=0$

$\therefore \int \frac{11-3x}{x^2+2x-3} = \int \frac{11-3x}{(x-1)(x+3)}\,dx$

Resolving $\frac{11-3x}{(x-1)(x+3)}$ into partial fraction

$\frac{11-3x}{(x+3)(x-1)} \equiv \frac{A}{(x+3)} + \frac{B}{(x-1)}$

$\frac{11-3x}{(x+3)(x-1)} \equiv \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}$

Equating the numerators

$11-3x = A(x-1)+B(x+3)$

Put $x=1$

$11-3(1) = A(1-1)+B(1+3)$

$8 = 4B$

$B = 2$

$\frac{11-3x}{(x+3)(x-1)} = \frac{-5}{(x+3)} + \frac{2}{(x-1)}$

$\int\left(\frac{11+3x}{(x+3)(x-1)}\right)dx = \int\left(\frac{-5}{(x+3)} + \frac{2}{(x-1)}\right)dx$

$= \int \frac{-5}{(x+3)}\,dx + \int \frac{2}{(x-1)}\,dx$

$= -5\int \frac{1}{(x+3)}\,dx + 2\int \frac{1}{(x-1)}\,dx$

$= -5\ln(x+3) + 2\ln(x-1)$

2) $\int \frac{4x-16}{x^2+2x-3}$

From the denominator,

$x^2-2x-3=0$

$x^2-3x+x-3=0$

$x(x-3)+1(x-3)=0$

$(x+1)(x-3)=0$

$\therefore \int \frac{4x-16}{x^2-2x-3} = \int \frac{4x-16}{(x-3)(x+1)}$

Resolving $\frac{4x-16}{(x-3)(x+1)}$ into partial fraction

$\frac{4x-16}{(x-3)(x+1)} \equiv \frac{A}{(x-3)} + \frac{B}{(x+1)}$

$\frac{4x-16}{(x-3)(x+1)} \equiv \frac{A(x+1)+B(x-3)}{(x-3)(x+1)}$

Equating the numerators

$4x-16 = A(x+1) + B(x-3)$

Put $x=3$

$4(3)-16 = A(3+1) + B(3-3)$

$-4 = 4A$

$A = -1$

Put $x=-1$

$4(-1)-16 = A(-1+1) + B(-1-3)$

$-20 = -4B$

$B = 5$

$\therefore \frac{4x-16}{(x-3)(x+1)} = \frac{-1}{(x-3)} + \frac{5}{(x+1)}$

$\int\left(\frac{4x-16}{(x-3)(x+1)}\right)dx = \int\left(\frac{-1}{(x-3)} + \frac{5}{(x+1)}\right)dx$

$= \int\frac{-1}{(x-3)}dx + \int\frac{5}{(x+1)}dx$

$= -1\int\frac{1}{(x-3)}dx + 5\int\frac{1}{(x+1)}dx$

$\therefore -1\ln(x-3) + 5\ln(x+1)$

3) $\int\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$

Resolving into partial fraction

$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$

$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3)+B(x+1)(x+3)+C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$

Equating the numerators

$2x^2-9x-35 = A(x-2)(x+3)+B(x+1)(x+3)+C(x+1)(x-2)$

Put $x=2$

$-45 = 15B$

$B = -3$

Put $x=-1$

$-24 = -6A$

$A = 4$

Put $x=-3$

$10 = 10C$

$C = 1$

$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{4}{(x+1)} - \frac{3}{(x-2)} + \frac{1}{(x+3)}$

$\int\left(\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\right)dx = \int\left(\frac{4}{(x+1)} - \frac{3}{(x-2)} + \frac{1}{(x+3)}\right)dx$

$= \int\frac{4}{(x+1)}dx - \int\frac{3}{(x-2)}dx + \int\frac{1}{(x+3)}dx$

$= 4\int\frac{1}{(x+1)}dx - 3\int\frac{1}{(x-2)}dx + 1\int\frac{1}{(x+3)}dx$

$= 4\ln(x+1) - 3\ln(x+2) + \ln(x+3)$