

$$\int \frac{11-3x}{x^2+2x-3}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

$$11-3x = Ax + 3A + Bx - B$$

$$11-3x = (3A-B) + (A+B)x$$

$$A+B = -3 \times 3 \quad \dots \textcircled{1}$$

$$3A-B = 11 \times 1 \quad \dots \textcircled{2}$$

$$3A + 3B = -9$$

$$3A - B = 11$$

$$AB = -20$$

$$B = -5 \quad (\text{substitute in equation 1})$$

$$A+B = -3$$

$$A-5 = -3$$

$$A = 2$$

$$\int \frac{11-3x}{x^2+2x-3} = \int \frac{2}{(x-1)} + \int \frac{B}{(x+3)}$$

$$= 2 \int \frac{1}{(x-1)} - 5 \int \frac{1}{(x+3)}$$

$$= 2 \ln(x-1) - 5 \ln(x+3)$$

$$\textcircled{2} \int \frac{11x-16}{x} = \int \frac{11-3x}{x}$$

$$2) \int \frac{4x-16}{x^2-2x-3} dx = \int \frac{4x-16}{(x+1)(x-3)} dx = \frac{A}{x+1} + \frac{B}{x-3}$$

$$\frac{4x-16}{(x+1)(x-3)} = \frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$$

$$4x-16 = A(x-3) + B(x+1)$$

$$4x-16 = (A+B)x + (B-3A)$$

$$A+B = 4 \quad \text{--- (1)}$$

$$B-3A = -16 \quad \text{--- (2)}$$

$$-3A - 3B = -12$$

$$-3A + B = -16$$

$$-4B = 4$$

$$B = -1 \quad (\text{Substitute in equation 1})$$

$$A - 1 = 4$$

$$A = 5$$

$$\int \frac{4x-16}{x^2-2x-3} dx = \int \frac{5}{x+1} + \int \frac{-1}{x-3}$$

$$= 5 \ln|x+1| - \ln|x-3|$$

$$3) \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$2x^2 - 9x - 35 = A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2)$$

$$2x^2 - 9x - 35 = Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$2x^2 - 9x - 35 = (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C)$$

Comparing coefficients:

$$A+B+C = 2 \quad \text{--- (1)}$$

$$A+4B-C = -9 \quad \text{--- (2)}$$

$$-6A+3B-2C = -35 \quad \text{--- (3)}$$

From (1) $A = 2 - B - C$ --- (4) (Substitute in 2 & 3)

$$2 - B - C + 4B - C = -9$$

$$-6(2 - B - C) + 3B - 2C = -35$$

$$3B - 2C = -11 \quad \times 9$$

$$9B + 4C = -23 \quad \times 3$$

$$27B - 18C = -99$$

$$-27B + 12C = -69$$

$$-30C = -30$$

$$C = 1$$

$$3B - 2C = -11$$

$$3B - 2 = -11$$

$$B = -3$$

$$A + B + C = 2$$

$$A - 3 + 1 = 2$$

$$A - 2 = 2$$

$$A = 4$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \int \frac{4}{(x+1)} - \int \frac{3}{(x-2)} + \int \frac{1}{(x+3)}$$

$$= 4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3|$$

For the correction to Number (2)

$$2) \int \frac{4x-16}{x^2-2x}$$

$$\frac{4x-16}{(x)(x-2)} = \frac{A}{x} + \frac{B}{x-2}$$

$$\frac{4x-16}{(x)(x-2)} = \frac{A(x-2) + Bx}{(x)(x-2)}$$

$$4x-16 = Ax - 2A + Bx$$

$$4x-16 = (A+B)x - 2A$$

Comparing coefficients

$$4 = A+B$$

$$-2A = -16$$

$$A = +8$$

$$A+B = 4$$

$$B = 4 - 8 = -4$$

$$\int \frac{4x-16}{x^2-2x} = \int \frac{8}{x} - \int \frac{4}{x-2}$$
$$= 8 \ln x - 4 \ln(x-2)$$

$$8 + B = 4$$

$$B = -8 + 4 = -4$$

$$\int \frac{4x-16}{x^2-2x} = \int \frac{8}{x} - \int \frac{4}{x-2}$$

$$= 8 \ln x - 4 \ln(x-2)$$