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Medicine and Surgery
191MHS01/293
MAT 104

$$1. \int \frac{11-3x}{x^2+2x-3} dx$$

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

Comparing; $11-3x = A(x+3) + B(x-1)$

At $x=1$; $f(1) \Rightarrow 11-3(1) = A(1+3) + B(1-1)$

$$8 = 4A$$

$$A = 2$$

At $x=-3$; $f(-3) \Rightarrow 11-3(-3) = A(-3+3) + B(-3-1)$

$$20 = -4B$$

$$B = -5$$

$$\frac{11-3x}{x^2+2x-3} = \frac{2}{x-1} - \frac{5}{x+3}$$

$$\int \frac{dx}{x^2+2x-3}$$

$$\int \frac{dx}{x-1}$$

$$\int \frac{dx}{x+3}$$

$$= 2 \int \frac{dx}{x-1} - 5 \int \frac{dx}{x+3}$$

$$\int \frac{11-3x}{x^2+2x-3} dx = 2 \ln(x-1) - 5 \ln(x+3) + k$$

(k is the constant of integration)

$$\int \frac{4x-16}{x^2-2x-3} dx$$

$$\frac{4x-16}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)}$$

$$\frac{4x-16}{(x-3)(x+1)} = \frac{A(x+1)+B(x-3)}{(x-3)(x+1)}$$

Comparing; $4x-16 = A(x+1) + B(x-3)$

At $x = -1$; $f(-1) = 4(-1) - 16 = A(-1+1) + B(-1-3)$
 $= -20 = -4B$

$$B = 5$$

At $x = +3$; $f(3) = 4(3) - 16 = A(3+1) + B(3-3)$

$$= -4 = 4A$$

$$A = -1$$

$$\frac{4x-16}{x^2-2x-3} = \frac{-1}{(x-3)} + \frac{5}{(x+1)}$$

$$\int \frac{4x-16}{x^2-2x-3} dx = \int \frac{-1}{(x-3)} dx + \int \frac{5}{(x+1)} dx$$

$$= -1 \int \frac{dx}{(x-3)} + 5 \int \frac{dx}{(x+1)}$$

$$\int \frac{4x-16}{x^2-2x-3} dx = -1 \int \frac{dx}{(x-3)} + 5 \int \frac{dx}{(x+1)}$$

$$= -1 \ln(x-3) + 5 \ln(x+1)$$

$$= 5 \ln(x+1) - \ln(x-3) + k$$

(k is the constant of integration)

3. $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

Comparing:

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+2) + C(x+1)(x-2)$$

• At $x = 2$

$$f(2) \Rightarrow 2(2^2) - 9(2) - 35 = A(2-2)(2+3) + B(2+1)(2+2) + C(2+1)(2-2)$$

$$-45 = 15B$$

$$B = -3$$

• At $x = -1$

$$f(-1) \Rightarrow 2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3) + B(-1+1)(-1+2) + C(-1+1)(-1-2)$$

$$-24 = -6A$$

$$A = 4$$

• At $x = 1$;

$$f(1) \Rightarrow 2(1)^2 - 9(1) - 35 = A(1-2)(1+3) + B(1+1)(1+2) + C(1+1)(1-2)$$

$$-42 = A(-4) + B(8) + C(-2)$$

$$-42 = -16 - 24 - 2C$$

$$2C = -16 - 24 + 42$$

$$2C = 2, \quad C = 1$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{(x+1)} - \frac{3}{(x-2)} + \frac{1}{x+3}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} dx + \int \frac{3}{x-2} dx + \int \frac{dx}{x+3}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = 4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3| + k$$

(k is constant of integration)