

$$1. \int \frac{11-3x}{x^2+2x-3} dx$$

$$x^2 + 2x - 3$$

$$x(x-1) + 3(x-1)$$

$$(x+3)(x-1)$$

$$\frac{A}{x+3} + \frac{B}{x-1} = \frac{11-3x}{(x+3)(x-1)}$$

$$\therefore A(x-1) + B(x+3) = 11-3x$$

$$Ax - A + Bx + 3B = 11-3x$$

$$(3B-A) + (A+B)x = 11-3x$$

$$3B-A = 11$$

$$A+B = -3$$

$$3B-11+B = -3$$

$$4B = 11-3 = 8$$

$$B = 2$$

$$A = -5$$

$$= \int \left( \frac{2}{x-1} - \frac{5}{x+3} \right) dx$$

$$= \int \frac{2}{x-1} dx - \int \frac{5}{x+3} dx$$

$$= 2 \ln|x-1| - 5 \ln|x+3| + C$$

$$2. \int \frac{4x-16}{x^2-2x-3} dx$$

$$\frac{4x-16}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3} \quad [x^2-2x-3 = (x+1)(x-3)]$$

$$\therefore 4x-16 = A(x-3) + B(x+1)$$

$$\text{Let } x = -1$$

$$4(-1)-16 = A(-1-3) + B(-1+1)$$

$$\therefore A = 5$$

$$\text{To find } B, \text{ Let } x = 3$$

$$4(3)-16 = A(3-3) + B(3+1)$$

$$\therefore B = -1$$

$$= \int \left[ \frac{5}{x+1} - \frac{1}{x-3} \right] dx$$

$$= 5 \int \frac{1}{x+1} dx - \int \frac{1}{x-3} dx$$

$$= 5 \ln|x+1| - \ln|x-3| + C$$

$$3. \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$2x^2-9x-35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

$$\dots = Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

Comparing coefficients;

$$2 = A + B + C \quad \text{--- (1)}$$

$$-9 = A + 4B - C \quad \text{--- (2)} \quad \therefore 9 = C - A - 4B$$

$$-35 = -6A + 3B - 2C \quad \text{--- (3)} \quad 35 = 6A - 3B + 2C$$

From (1) & (2)

$$2C - 3B = 11; \text{ adding this eqn to (3), we have}$$

$$-6A = -24$$

$$A = 4$$

$$\therefore B + C = -2$$

$$C = -2 - B$$

$$2[-2 - B] + 3B = 11$$

$$-4 - 2B - 3B = 11$$

$$-5B = 15$$

$$\therefore B = -3$$

$$C = -2 - (-3)$$

$$C = 1$$

$$\therefore A = 4, B = -3, C = 1$$

$$= \int \left[ \frac{4}{(x+1)} - \frac{3}{(x-2)} + \frac{1}{(x+3)} \right] dx$$

$$= 4 \int \frac{1}{(x+1)} dx - 3 \int \frac{1}{(x-2)} dx + \int \frac{1}{(x+3)} dx$$

$$= 4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3| + C$$