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DEPARTMENT: MBBS
COURSE: MATHS 104
MATRIC NO: 19/MHSOL/385

1. $\int \frac{11-3x}{x^2+2x-3} = \int \frac{11+3x}{x^2+2x-3}\,dx$

The denominator $= x^2+2x-3$

$=(x^2+3x)-(x-3)=0$

$x(x+3)-(x+3)=0$

$=(x-1)(x+3)=0$

$= \int \frac{11-3x}{(x-1)(x+3)}\,dx$

$\therefore \frac{11-3x}{(x+3)(x-1)} = \frac{A}{(x+3)} + \frac{B}{(x-1)}$

$\frac{11-3x}{(x+3)(x-1)} = \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}$

Equate the numerator

$\therefore 11-3x = Ax - A + Bx + 3B$

Substitute $x=1$

$11-3(1) = A(1) - A + B(1) + 3B$

$11-3 = A - A + B + 3B$

$\frac{8}{4} = \frac{4B}{4} \quad \therefore B = 2$

Substitute $x=-3$

$11-3x = Ax - A + Bx + 3B$

$11-3(-3) = A(-3) - A + B(-3) + 3B$

$11+9 = -3A - A - 3B + 3B$

$\frac{20}{-4} = \frac{-4A}{-4} \quad \therefore A = -5$

$$\frac{11-3x}{(x+3)(x-1)} = \frac{-5}{(x+3)} + \frac{2}{(x-1)}$$

$$\therefore \int \frac{11-3x}{(x+3)(x-1)}\,dx = \int \left(\frac{-5}{(x+3)} + \frac{2}{(x-1)}\right) dx$$

$$= \int \frac{-5}{x+3}\,dx + \int \frac{2}{(x-1)}\,dx$$

$$= -5\int \frac{1}{(x+3)}\,dx + 2\int \frac{1}{(x-1)}\,dx$$

$$= -5\ln(x+3) + 2\ln(x-1)$$

2 $\displaystyle\int \frac{4x-16}{x^2-2x-3}$

The denominator $= x^2-2x-3$

$$(x^2-3x)+(x-3) = 0$$

$$x(x-3)+(x-3) = 0$$

$$(x-3)(x+1) = 0$$

$$= \int \frac{4x-16}{(x-3)(x+1)}\,dx$$

$$\therefore \frac{4x-16}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)}$$

$$\frac{4x-16}{(x-3)(x+1)} = \frac{A(x+1)+B(x-3)}{(x-3)(x+1)}$$

Equate the numerator

$$4x-16 = Ax + A + Bx - 3B$$

Substitute $x = 3$

$$4(3)-16 = A(3)+A+B(3)-3B$$

$$12-16 = 3A + A + 0$$

$$\frac{-4}{4} = \frac{4A}{4}$$

$$\therefore A = -1$$

Substitute $x = -1$

$4x - 16 = A(x+1) + B(x-3)$

$4(-1) - 16 = A(-1+1) + B(-1-3)$

$-20 = 0 - 4B$

$\frac{-20}{-4} = \frac{-4B}{-4} \quad \therefore B = 5$

$\frac{4x-16}{(x-3)(x+1)} = \frac{-1}{x-3} + \frac{5}{x+1}$

$\therefore \int \left(\frac{4x-16}{(x-3)(x+1)}\right) dx = \int \left(\frac{-1}{x-3} + \frac{5}{x+1}\right) dx$

$= \int \frac{-1}{x-3}\, dx + \int \frac{5}{x+1}\, dx$

$= -1 \int \frac{1}{(x-3)}\, dx + 5 \int \frac{1}{(x+1)}\, dx$

$= -1 \ln(x-3) + 5 \ln(x+1)$

3 $\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}$

Resolving into partial fraction

$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$

$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$

Equate the numerators

$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$

Substitute $x = 2$

$2(2)^2 - 9(2) - 35 = B(2+1)(2+3)$

$\frac{-45}{+15} = \frac{+15B}{+15}$

$\therefore B = -3$

Substitute $x = -1$

$2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3)$

$\frac{-24}{6} = \frac{6A}{6}$

$\therefore A = -4$

Substitute $x = -3$

$2(-3)^2 - 9(-3) - 35 = C(-3+1)(-3-2)$

$\frac{10}{10} = \frac{10C}{10}$

$\therefore C = 1$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{(x+1)} - \frac{3}{(x-2)} + \frac{1}{(x+3)}$$

$$\int \left(\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}\right) dx = \int \left(\frac{4}{(x+1)} \frac{-3}{(x-2)} + \frac{1}{(x+3)}\right) dx$$

$$= \int \frac{4}{(x+1)} dx - \int \frac{3}{(x-2)} dx + \int \frac{1}{(x+3)} dx$$

$$4 \int \frac{1}{(x+1)} dx - 3 \int \frac{1}{(x-2)} dx + \int \frac{1}{(x+3)} dx$$

$$= 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)$$