13 $\int \frac{11-3x}{x^2+2x-3}$

Solution

$x^2+2x-3 = x^2+3x-x-3$

$= x(x+3)-1(x+3)$

$= (x-1)(x+3)$

$\Rightarrow \int \frac{11-3x}{x^2+2x-3} = \int \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$

$= \frac{A(x-1)(x+3)+B(x-1)(x+3)}{(x-1)(x+3)}$

$11-3x = A(x+3)+B(x-1)$

$11-3x = Ax+3A+Bx-B$

$11-3x = Ax+Bx+3A-B$

$11-3x = (A+B)x+(3A-B)$

Comparing

$A+B = -3$ ——— (i) ——— 13

$3A-B = 11$ ——— (ii) ——— 11

$-\ 3A+3B = -9$ ——— (iii)

$3A-B = 11$ ——— (iv)

$4B = -20$

$B = -20/4 \therefore B = -5$

Sub 5 for B in eqn (i)

$A+B = -3$

$A-5 = -3$

$A = -3+5$

$A = 2$

$$\therefore \int \frac{A}{x-1} + \int \frac{B}{x+5} = \int \frac{11-3x}{x^2+2x-5}\,dx$$

$$= \int \frac{2\,dx}{x-1} + \int \frac{-5\,dx}{x+5} = \int \frac{11-3x}{x^2+2x-5}\,dx$$

$u = x-1$ $\quad$ $u = x+5$

$du = dx$ $\quad$ $du = dx$

$$= \int \frac{2\,du}{u} \quad \int \frac{-5\,du}{u}$$

$$= 2\ln u \quad -5\ln u$$

$$\therefore \int \frac{11-3x}{x^2+2x-5} = 2\ln u + (-5\ln u) = 2\ln u - 5\ln u$$

$$= 2\ln(x-1) - 5\ln(x+5)$$

2) $\displaystyle\int \frac{4x-16}{x^2-2x-5}$

Solution

$$x^2-2x-5 = x^2-5x+x-5$$

$$= x(x-5)+1(x-5)$$

$$= (x+1)(x-5)$$

$$\Rightarrow \int \frac{4x-16}{(x+1)(x-5)} = \frac{A}{(x+1)} + \frac{B}{(x-5)}$$

$$\frac{A(x-5)+B(x+1)}{(x+1)(x-5)}$$

$$4x-16 = A(x-5)+B(x+1)$$

$$4x-16 = Ax-5A+Bx+B$$

$$4x-16 = Ax+Bx-5A+B$$

$$4x-16 = (A+B)x + (-5A+B)$$

Comparing

$$A+B = 4$$

$$-5A+B = -16$$

$$4A = 20$$

$$A = 5$$

Sub 5 for A in eqn 1

$A+B=4$

$5+B=4$

$B=4-5$

$B=-1$

$$\int\frac{A}{x+1}+\int\frac{B}{x-5}=\int\frac{4x+6}{x^2-2x-5}$$

$$\int\frac{5\,dx}{x+1}+\int\frac{-1\,dx}{x-5}$$

Let $u=x+1$ $\quad u=x-5$

$du=dx$ $\quad du=dx$

$$\int\frac{5\,du}{u} \qquad \int\frac{-1\,du}{u}$$

$=5\ln|u|$ $\quad =-\ln|u|$

$$\therefore \int\frac{4x+6}{x^2-2x-5}=5\ln|u|-\ln|u|=5\ln(x+1)-\ln(x-5)$$

3) $\dfrac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}=\frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{x+3}$$

$$=\frac{A(x-2)(x+3)+B(x+1)(x+3)+C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$=A(x-2)(x+3)+B(x+1)(x+3)+C(x+1)(x-2)$

$=A(x^2+x-6)+B(x^2+4x+3)+C(x^2-x-2)$

$=Ax^2-Ax-6A+Bx^2+4Bx+3B+Cx^2-Cx-2C$

$=Ax^2+Bx^2+Cx^2-Ax+4Bx-Cx-6A+3B-2C$

$2x^2-9x-35=(A+B+C)x^2+(A+4B-C)x+(-6A+3B-2C)$

Comparing $\Rightarrow$

$A+B+C=2$ —— (1)

$A+4B-C=-9$ —— (2)

$-6A+3B-2C=-35$ —— (3)

$A+B+C=2$

$A+4B-C=-9$

$-3B+2C=11$ $\therefore$ $3B-2C=-11$ —— (iv)

(Put in eqn 3)

$-6A+3B-2C=-35$

$-6A-11=-35$

$-6A=-35+11$

$-6A=+24$ $\quad A=4$

$4+B+C=2 \quad \therefore \quad B+C=-2$

$4+4B+C=-9 \quad 4B+C=-13$

$B+C=-2$ — (5)

$4B+C=-13$ — (6)

$5B=-15$

$B=-3$

Sub -3 for B in eqn (5)

$B+C=-2$

$-3+C=-2$

$C=-2+3$

$C=1$

$A=4,\ B=-3\ \ C=1$

$$\int \frac{2x^2-9x-55}{(x+1)(x-2)(x+3)}\,dx = \int \frac{4\,dx}{x+1} + \int \frac{-3\,dx}{x-2} + \int \frac{dx}{x+3}$$

Let $u=x+1 \quad u=x-2 \quad u=x+3$

$du=dx$

$$= \int \frac{4\,du}{u} \quad = \int \frac{-3\,du}{u} \quad = \int \frac{du}{u}$$

$$= 4\ln u \quad = -3\ln u \quad = \ln u$$

$$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = 4\ln u - 3\ln u + \ln u$$

$$= \underline{\underline{4\ln(x+1) - 3\ln(x-2) + \ln(x+3)}}$$