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LEVEL: 100

MATRIC NO: 19/MHSO1/148

1. $\frac{11-3x}{x^2+2x-3}$

$$\Rightarrow \int \frac{11-3x}{x^2+2x-3} dx$$

From the denominator,

$$x^2+2x-3=0$$

$$x^2+3x-x-3=0$$

$$x(x+3)-1(x+3)=0$$

$$(x-1)(x+3)=0$$

$$\therefore \int \frac{11-3x}{x^2+2x-3} dx = \int \frac{11-3x}{(x-1)(x+3)} dx$$

Resolving $\frac{11-3x}{(x-1)(x+3)}$ into partial fractions

$$\frac{11-3x}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$$

$$\frac{11-3x}{(x+3)(x-1)} = \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}$$

Equating the numerators

$$11-3x = A(x-1)+B(x+3)$$

Put $x=1$

$$11-3(1) = A(1-1)+B(1+3)$$

$$8 = 4B$$

$$B = 2$$

Put $x=-3$

$$11-3(-3) = A(-3-1)+B(-3+3)$$

$$20 = -4A$$

$$A = -5$$

$$\therefore \frac{11-3x}{(x+3)(x-1)} = \frac{-5}{x+3} + \frac{2}{x-1}$$

$$\int \left(\frac{11-3x}{(x+3)(x-1)} \right) dx = \int \left(\frac{-5}{x+3} + \frac{2}{x-1} \right) dx$$

$$= \int \frac{-5}{x+3} dx + \int \frac{2}{x-1} dx$$

$$= -5 \int \frac{1}{x+3} dx + 2 \int \frac{1}{x-1} dx$$

$$\Rightarrow -5 \ln(x+3) + 2 \ln(x-1)$$

2. $\int \frac{4x-16}{x^2-2x-3}$

From the denominator,

$$x^2-2x-3=0$$

$$x^2-3x+x-3=0$$

$$x(x-3)+1(x-3)=0$$

$$(x-3)(x+1)=0$$

$$\therefore \int \frac{4x-16}{x^2-2x-3} = \int \frac{4x-16}{(x-3)(x+1)}$$

Resolving $\frac{4x-16}{(x-3)(x+1)}$ into partial fractions

$$\frac{4x-16}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$\frac{4x-16}{(x-3)(x+1)} = \frac{A(x+1)+B(x-3)}{(x-3)(x+1)}$$

Equating the numerators

$$4x-16 = A(x+1)+B(x-3)$$

Put $x=3$

$$4(3)-16 = A(3+1)+B(3-3)$$

$$-4 = 4A$$

$$A = -1$$

Put $x=-1$

$$4(-1)-16 = A(-1+1)+B(-1-3)$$

$$-20 = -4B$$

$$B = 5$$

$$\therefore \frac{4x-16}{(x-3)(x+1)} = \frac{-1}{x-3} + \frac{5}{x+1}$$

$$\int \left(\frac{4x-16}{(x-3)(x+1)} \right) dx = \int \left(\frac{-1}{x-3} + \frac{5}{x+1} \right) dx$$

$$= \int \frac{-1}{x-3} dx + \int \frac{5}{x+1} dx$$

$$= -1 \int \frac{1}{x-3} dx + 5 \int \frac{1}{x+1} dx$$

$$\Rightarrow -1 \ln(x-3) + 5 \ln(x+1)$$

$$3. \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}$$

Resolving into partial fraction

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

Equating the numerators

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

$$\text{Put } x = 2$$

$$-45 = 15B$$

$$B = -3$$

$$\text{Put } x = -1$$

$$-24 = -6A$$

$$A = 4$$

$$\text{Put } x = -3$$