LOLOMARI IBITEIN DEBORAH

MEDICINE AND SURGERY

19/MHS01/287 COURSE: MAT 104

Assignment on integration of partial fractions

1 $\int \frac{11-3x}{(x^2+2x-3)}\,dx$ (x^2+2x-3)

$= \int \frac{11-3x}{(x-1)(x+3)}$ $= x^2+3x-x-3$

$= x(x+3)-1(x+3)$

$= (x-1)(x+3)$

$= \frac{11-3x}{(x-1)(x+3)}$

$\Rightarrow \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$ (multiply both sides by $(x-1)(x+3)$)

$\Rightarrow 11-3x = A(x+3) + B(x-1)$

[Opening the brackets,]

$\Rightarrow Ax+3A+Bx-B = 11-3x$

$\Rightarrow Ax+Bx-B+3A = 11-3x$

$\Rightarrow x(A+B) + (3A-B) = 11-3x$ (by elimination)

$A+B = -3$ --- i

$3A-B = 11$ --- ii

$\frac{4A}{4} = \frac{8}{4}$

$A = 2$

Sub for A in eqn i

$2+B = -3$

$B = -3-2$

$\therefore B = -5$

$\therefore \int \frac{2}{x-1}dx + \int \frac{-5}{x+3}dx = \int \frac{11-3x}{x^2+2x-3}dx$

$\Rightarrow \int \frac{2dx}{x-1} + \int \frac{-5dx}{x+3} = \int \frac{11-3x}{x^2+2x-3}dx$

Let $u_1 = x-1$ $u_2 = x+3$

$du = dx$ $du = dx$

$\Rightarrow \int \frac{2du}{u}$ $\Rightarrow \int \frac{-5du}{u}$

$= 2\ln u$ $= -5\ln u$

$\Rightarrow 2\ln(x-1) - 5\ln(x+3)$

2 $\int \frac{4x-16}{(x^2-2x-3)} dx$

$\Rightarrow \frac{4x-16}{x^2-2x-3}$

$\rightarrow x^2-2x-3$

$= x^2-3x+x-3$

$= x(x-3)+1(x-3)$

$= (x+1)(x-3)$

$\Rightarrow \frac{4x-16}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$

multiply both sides by $(x+1)(x-3)$

$\Rightarrow 4x-16 = A(x-3)+B(x+1)$

[opening brackets,]

$\Rightarrow Ax-3A+Bx+B = 4x-16$

$\Rightarrow Ax+Bx+B-3A = 4x-16$

$\Rightarrow (A+B)x+(B-3A) = 4x-16$

by ratio compare both eqns

$A+B = 4 \quad ---\ 1$

$-3A+B = -16 \quad ---\ 11$

By elimination

$\frac{-4A}{-4} = \frac{-20}{-4}$

$A = 5$

Sub for A in eqn 1

$5+B = 4$

$B = 4-5$

$B = 1$

$\therefore \int \frac{4x-16}{x^2-2x-3} dx = \int \frac{5}{x+1} dx + \int \frac{1}{x-3} dx$

$\int \frac{4x-16}{x^2-2x-3} dx = \int \frac{5dx}{x+1} + \int \frac{dx}{x-3}$

Let $u = x+1$ $\quad u = x-3$

$du = dx$ $\quad du = dx$

$= \int \frac{5du}{u}$ $\quad = \int \frac{du}{u}$

$= 5\ln u + \ln u$

$\therefore \int \frac{4x-16}{x^2-2x-3} dx = 5\ln(x+1) + \ln(x-3)$

3 $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx$

$\Rightarrow \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$

multiply both sides by $(x+1)(x-2)(x+3)$

$\Rightarrow 2x^2-9x-35 = A(x-2)(x+3)+B(x+1)(x+3)+C(x+1)(x-2)$

$\Rightarrow 2x^2-9x-35 = A(x^2+3x-2x-6)+B(x^2+3x+x+3)+C(x^2-2x+x-2)$

$\Rightarrow 2x^2-9x-35 = A(x^2+x-6)+B(x^2+4x+3)+C(x^2-x-2)$

$\Rightarrow 2x^2-9x-35 = Ax^2+Ax-6A+Bx^2+4Bx+3B+Cx^2-Cx-2C$

$\Rightarrow 2x^2-9x-35 = Ax^2+Bx^2+Cx^2+Ax+4Bx-Cx-6A+3B-2C$

$\Rightarrow 2x^2-9x-35 = (A+B+C)x^2+(A+4B-C)x+(-6A+3B-2C)$

Compare both sides of the eqn.

$A+B+C = 2 \quad \cdots I$

$A+4B-C = -9 \quad \cdots II$

$-6A+3B-2C = -35 \quad \cdots III$

Let $A = 2-B-C \quad \cdots IV$

Sub for A in eqn II and III

" $2-B-C+4B-C = -9$

$2+3B-2C = -9$

$\Rightarrow 3B-2C = -11 \quad \cdots V$

" $-6(2-B-C)+3B-2C = -35$

$-12+6B+6C+3B-2C = -35$

$\Rightarrow 9B+4C = -23$

By elimination

$3B-2C = -11 \times 3$

$9B+4C = -23$

$\Rightarrow 9B-6C = -33$ SSS

$9B-4C = -23$ DSA

$\frac{10C}{10} = \frac{10}{10}$

$C = 1$

Sub for C in eqn V

$3B-2 = -11$

$3B = -11+2$

$\frac{3B}{3} = \frac{-9}{3}$

$B = -3$

Sub for B & C in eqn IV

$A = 2-B-C$

$A = 2-(-3)-1$

$A = 4$

$\therefore A=4, B=-3, C=1$

$\therefore \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}dx = \int \frac{4}{x+1}dx - \int \frac{3}{x-2}dx + \int \frac{1}{x+3}dx$

$\int \frac{4dx}{(x+1)} - \int \frac{3dx}{x-2} + \int \frac{dx}{x+3} = \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}dx$

$u = x+1 \quad u = x-2 \quad u = x+3$

$du = dx \quad du = dx \quad du = dx$

$= \int \frac{4du}{u} - \int \frac{3du}{u} + \int \frac{du}{du}$

$\Rightarrow 4\ln u \Rightarrow 3\ln u \Rightarrow \ln u$

$\therefore \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}dx = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3)$