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Course: MAT 104

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Solution to Assignment

1) $\int \frac{11-3x}{(x^2+2x-3)} dx$

from the denominator;

$x^2+2x-3=0$; $x^2+3x-x-3=0$

$x(x+3)-1(x+3)=0$

$(x-1)(x+3)=0$

$\therefore \int \frac{11-3x}{x^2+2x-3} dx = \int \frac{11-3x}{(x-1)(x+3)} dx$

Resolving $\frac{11-3x}{(x-1)(x+3)}$ into partial fraction

~~$\frac{11-3x}{(x+3)(x-1)} = A(x-1)+B(x+3)$~~

$\frac{11-3x}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$

$\frac{11-3x}{(x+3)(x-1)} = \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}$

Equating the numerators.

$11-3x = A(x-1)+B(x+3)$

$\rightarrow$ put $x=1$

$f(1) \Rightarrow 11-3(1) = B(1+3)$

$\therefore \frac{8}{2} = 4B$

$\frac{\cancel{8}}{4} = \frac{\cancel{4}B}{\cancel{4}}$ $\therefore B = 2$

→ put $x = -3$

$f(-3) \Rightarrow 11 - 3(-3) = A(-3-1)$

$\therefore 20 = -4A$ $\therefore \frac{-\cancel{4}A}{-\cancel{4}} = \frac{20}{-4} = -5$

$\therefore A = -5$

Substitute the value of A & B into the equation.

$$\frac{11-3x}{(x+3)(x-1)} = \frac{-5}{x+3} + \frac{2}{x-1}$$

$$\int \frac{11-3x}{(x+3)(x-1)} dx = \int \frac{-5}{x+3} dx + \int \frac{2}{x-1} dx$$

let $u = x + 3$

$\frac{du}{dx} = 1$

$dx = \frac{du}{1}$

$$\int \frac{-5}{x+3} dx = \int \frac{-5}{u} \cdot du = -5 \int \frac{1}{u} du = -5 \ln u = -5 \ln(x+3)$$

let $u = x - 1$

$\frac{du}{dx} = 1$ $\therefore dx = \frac{du}{1}$

$$\int \frac{2}{x-1} dx = \int \frac{2}{u} \cdot du = 2 \int \frac{1}{u} du = 2 \ln u = 2 \ln(x-1)$$

$$\therefore \int \frac{11-3x}{(x+3)(x-1)} = 5\ln(x+3) + 2\ln(x-1)$$

$$\Rightarrow \int \frac{11-3x}{(x+3)(x-1)} = -5\ln(x+3 + 2\ln(x-1) + C$$

2.) $\int \frac{4x-16}{(x^2-2x-3)} dx$

from the denominator $\therefore x^2-2x-3=0$

$x^2-3x+x-3=0 \therefore x(x-3)+1(x-3)=0$

$\therefore (x-3)(x+1)=0$

$$\therefore \int \frac{4x-16}{x^2-2x-3} = \int \frac{4x-16}{(x-3)(x+1)}$$

Resolving $\int \frac{4x-16}{(x-3)(x+1)}$ into partial fraction:

$$\int \frac{4x-16}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)}$$

$$\therefore \frac{4x-16}{(x-3)(x+1)} = \frac{A(x+1)}{(x-3)} + \frac{B(x-3)}{(x+1)}$$

equating the numerators

$4x-16 = A(x+1) + B(x-3)$

$\Rightarrow$ put $x=-1$

$f(-1) \Rightarrow 4(-1)-16 = B(-1-3)$

$-20 = -4B$

$\frac{-20}{-4} = \frac{-4B}{-4} \quad \therefore B = 5$

→ put $x = 3$

$f(3) \Rightarrow 4(3) - 16 = A(3+1)$

$= -4 = 4A$

$\frac{-4}{4} = \frac{4A}{4} \quad \therefore A = -1$

Substitute the values of A & B into the equation:

$$\frac{4x-16}{(x-3)(x+1)} = \frac{-1}{(x-3)} + \frac{5}{(x+1)}$$

$$\int \frac{4x-16}{(x-3)(x+1)} = \int \frac{-1}{(x-3)} dx + \int \frac{5}{(x+1)} dx$$

let $u = x - 3$

$\frac{du}{dx} = 1 \therefore dx = \frac{du}{1}$

$$\int \frac{-1}{(x-3)} dx = \int \frac{-1}{u} \cdot dx = -1 \int \frac{1}{u} dx = -1 \ln u = -1 \ln (x-3)$$

let $u = x + 1$

$\frac{du}{dx} = 1 \therefore dx = \frac{du}{1}$

$$\int \frac{5}{(x-3)} dx = \int \frac{5}{u} \cdot dx = 5 \int \frac{1}{u} dx = 5 \ln u = 5 \ln (x+1)$$

$$\therefore \int \frac{4x-16}{(x-3)(x+1)} = -1 \ln (x-3) + 5 \ln (x+1) + C$$

3.) $\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}\,dx$; Resolving into partial fraction:

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} \equiv \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} \equiv \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

equating the numerator:

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

put $x = 2$

$$f(2) \Rightarrow 2(2)^2 - 9(2) - 35 = B(2+1)(2+3)$$

$$-45 = 15B$$

$$\frac{-\cancel{45}^{3}}{\cancel{18}} = \frac{\cancel{18}B}{\cancel{18}} \quad \therefore B = -3$$

put $x = -1$

$$f(1) \Rightarrow 2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3)$$

$$-24 = -6A$$

$$\frac{-\cancel{24}}{\cancel{-6}}^{4} = \frac{\cancel{-6}A}{\cancel{-6}}$$

$$\therefore A = 4$$

put $x = -3$

$$f(-3) \Rightarrow 2(-3)^2 - 9(-3) - 35 = C(-3+1)(-3-2)$$

$$10 = 10C$$

$$\frac{\cancel{10}^{1}}{\cancel{10}} = \frac{\cancel{10}C}{\cancel{10}}$$

$$\therefore C = 1$$

Substitute the values of A, B & C in the equation

$$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} + \frac{(-3)}{x-2} + \frac{1}{x+3}$$

$$\int \frac{2x^2-9x-35}{(x-1)(x-2)(x+3} = \int \frac{4}{x+1}dx - \int \frac{3}{x-2}dx + \int \frac{1}{x+3}dx$$

let $u = x+1$

$\frac{du}{dx} = 1 \therefore dx = \frac{du}{1}$

$$\int \frac{4}{x+1}dx = \int \frac{4}{u} \cdot du = 4\int \frac{1}{u}du = 4\ln u = 4\ln(x+1)$$

let $u = x-2$

$\frac{du}{dx} = 1 \therefore dx = \frac{du}{1}$

$$\int \frac{3}{x-2}dx = \int \frac{3}{u} \cdot du = 3\int \frac{1}{u}du = 3\ln u = 3\ln(x-2)$$

let $u = x+3$

$\frac{du}{dx} = 1$

$dx = \frac{du}{1} \therefore \int \frac{1}{x+3}dx = \int \frac{1}{u} \cdot du = 1\int \frac{1}{u}du = 1\ln u = 1\ln(x+3)$

$$\therefore \int \frac{(2x^2-9x-35)}{(x+1)(x-2)(x+3)} = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3)$$

$$\int \frac{(2x^2-9x-35)}{(x+1)(x-2)(x+3)} = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3 + C.$$

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