Mat assignment

1) $\int \frac{11-3x}{x^2+2x-3}\,dx$

x^2+2x-3

$x^2+3x-x-3$

$x(x+3)-1(x+3)$

$(x-1)(x+3)$

Soln

$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)}$

$= \frac{A}{(x-1)} + \frac{B}{(x+3)} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$

$A(x+3)+B(x-1) = 11-3x$

$Ax+3A+Bx-B = 11-3x$

$(A+B)x+(3A-B) = 11-3x$

Comparing $A+B = -3 \quad \cdots (1)$

$3A-B = 11 \quad \cdots (2)$

$\frac{4A}{4} = \frac{8}{4}$

$\therefore A = 2$

Put A in eqn (1)

$A+B = -3$

$2+B = -3$

$B = -3-2 \Rightarrow B = -5$

We can now write

$\int \frac{2}{x-1}\,dx + \int \frac{-5}{x+3}\,dx = \int \frac{11-3x}{x^2+2x-3}$

Let $v = x-1$

$dv = dx$

$v = x+3$

$dv = dx$

We can now write

$$\int \frac{2}{x-1}\,dx + \int \frac{-5}{x+3}\,dx = \int \frac{11-3x}{x^2+2x-3}$$

Let $u = x-1$ $\quad u = x+3$

$du = dx$ $\quad du = dx$

$\Rightarrow \int \frac{2\,du}{u}$ $\quad \Rightarrow \int \frac{-5\,du}{x+3u}$

$\therefore 2\ln u \Rightarrow -5\ln u$

$\Rightarrow 2\ln(x-1) - 5\ln(x+3) + C$

27 $\int \frac{4x-16}{(x^2-2x-3)}\,dx$

Solution

$x^2 - 2x - 3$

$x^2 - 3x + x - 3$

$x(x-3) + 1(x-3)$

$(x+1)(x-3)$

$\int \frac{4x-16}{(x^2-2x-3)} \Rightarrow \int \frac{4x-16}{(x+1)(x-3)}$

$= \frac{A}{x+1} + \frac{B}{x-3} \Rightarrow \frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$

$A(x-3) + B(x+1) = 4x - 16$

$Ax - 3A + Bx + B = 4x - 16$

$(B+A)x + (B-3A) = 4x - 16$

(comparing sides) $B + A = 4 \cdots ①$

$B - 3A = -16 \cdots ②$

$\frac{4A}{4} = \frac{20}{4}$ $\quad \therefore A = 5$

Put (A) in eqn ①

$B + 5 = 4$

$B = 4 - 5$ $\quad \therefore B = -1$

We can now write

$$\int \frac{5}{x+1}\,dx + \int \frac{-1}{x-3}\,dx = \int \frac{4x-16}{x^2-2x-3}\,dx$$

Let $u = x+1$ $\quad$ $v = x-3$

$du = dx$ $\quad$ $dv = dx$

$\Rightarrow \int \frac{5\,du}{u}$ $\quad$ $\Rightarrow \int \frac{-5\,dv}{v}$

$\therefore 5\ln v \Rightarrow -\ln u$

$\therefore 5\ln(x+1) - \ln(x-3) + C$

3) $\int \frac{(2x^2 - 9x - 35)}{(x+1)(x-2)(x+3)}\,dx$

Solution

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x+3)(x-2)}$$

$2x^2 - 9x - 35 = A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2)$

$2x^2 - 9x - 35 = Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$

$2x^2 - 9x - 35 = Ax^2 + Bx^2 + Cx^2 + Ax + 4Bx - Cx - 6A + 3B - C$

$2x^2 - 9x - 35 = (A + B + C)x^2 + (A + 4B - C)x + (-6A + 3B - 2C)$

comparing $\therefore A + B + C = 2 \cdots (1)$

$A + 4B - C = -9 \cdots (2)$

$-6A + 3B - 2C = -35 \cdots (3)$

$A + B + C = 2$

$A + 4B - C = -9$

$-3B + 2C = 11 \quad \therefore 3B - 2C = -11$

Put in eqn 3

$\therefore -6A(-11) = -35$

$-6A = -35 + 11 = -24$

$\frac{-6A}{-6} = \frac{-24}{-6}$

$A = 4$

$\therefore \quad A+B+C = 2 \Rightarrow B+C = 2-4 \Rightarrow B+C = -2$

$4+4B-C = -9 \quad 4B-C = -9-4 \Rightarrow 4B-C = -B$

$B+C = -2$

$\underline{4B-C = -13}$

$\frac{5B}{5} = \frac{-15}{5} \quad \therefore B = -3$

Finally $-3+C = -2$

$C = -2+3$

$C = 1$

$\therefore A=4, B=-3, C=1$

$$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx = \int \frac{4\,dx}{x+1} + \int \frac{3\,dx}{x-2} + \int \frac{dx}{x+3}$$

$x+1 = u \qquad u = x-2, \quad u = x+3$

$du = dx \qquad du = dx \qquad du = du$

$$\Rightarrow 4\int \frac{du}{u} - 3\int \frac{du}{u} \qquad \int \frac{du}{u}$$

$\Rightarrow 4\ln u - 3\ln u + \ln u$

$4\ln(x+1) - 3\ln(x-2) + \ln(x+3)$