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MATRIC NUMBER: 19/MHS01/320
COURSE: MAT 104

ASSIGNMENT

1. $\int \frac{11 - 3x}{(x^2 + 2x - 3)} dx$

Solution

$\int \frac{11 - 3x}{(x^2 + 2x - 3)} dx$

From the denominator,

$x^2 + 2x - 3 = 0$

$x^2 + 3x - x - 3 = 0$

$x(x + 3) - 1(x + 3) = 0$

$(x - 1)(x + 3) = 0$

$\therefore \int \frac{11 - 3x}{x^2 + 2x - 3} dx = \int \frac{11 - 3x}{(x-1)(x+3)} dx$

Resolving $\frac{11 - 3x}{(x-1)(x+3)}$ into partial fraction.

$\frac{11 - 3x}{(x+3)(x-1)} \equiv \frac{A}{x+3} + \frac{B}{x-1}$

$\frac{11 - 3x}{(x+3)(x-1)} \equiv \frac{A(x-1) + B(x+3)}{(x+3)(x-1)}$

Equating the numerators

$11 - 3x = A(x-1) + B(x+3)$

→ Put $x = 1$

$f(1) \Rightarrow 11 - 3(1) = B(1 + 3)$

$8 = 4B$

$B = \frac{8}{4}$

$B = 2$

→ Put $x = -3$

$f(-3) \Rightarrow 11 - 3(-3) = A(-3-1)$

$20 = A(-4)$

$A = \frac{20}{-4}$

$A = -5$

Substitute the value of A and B in the equation

$$\frac{11-3x}{(x+3)(x-1)} = \frac{-5}{x+3} + \frac{2}{x-1}$$

$$\int \frac{11-3x}{(x+3)(x-1)}\,dx = \int \frac{-5}{x+3}\,dx + \int \frac{2}{x-1}\,dx$$

Let $u = x + 3$

$\frac{du}{dx} = 1$

$dx = \frac{du}{1}$

$$\int \frac{-5}{x+3}\,dx = \int \frac{-5}{u} \cdot du = -5\int \frac{1}{u}\,du = -5\ln u = -5\ln(x+3)$$

Let $u = x - 1$

$\frac{du}{dx} = 1$

$dx = \frac{du}{1}$

$$\int \frac{2}{x-1}\,dx = \int \frac{2}{u} \cdot du = 2\int \frac{1}{u}\,du = 2\ln u = 2\ln(x-1)$$

$$\therefore \int \frac{11-3x}{(x+3)(x-1)} = -5\ln(x+3) + 2\ln(x-1)$$

$$\Rightarrow \int \frac{11-3x}{(x+3)(x-1)} = -5\ln(x+3) + 2\ln(x-1) + C$$

2. $$\int \frac{4x-16}{(x^2-2x-3)}\,dx$$

Solution

From the denominator

$x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$(x-3)(x+1) = 0$

$$\therefore \int \frac{4x-16}{x^2-2x-3} = \int \frac{4x-16}{(x-3)(x+1)}$$

Resolving $\frac{4x-16}{(x-3)(x+1)}$ into partial fraction.

$$\frac{4x-16}{(x-3)(x+1)} \equiv \frac{A}{(x-3)} + \frac{B}{(x+1)}$$

$$\frac{4x-16}{(x-3)(x+1)} \equiv \frac{A(x+1) + B(x-3)}{(x-3)(x+1)}$$

Equating the numerators

$4x - 16 = A(x+1) + B(x-3)$

$\rightarrow$ Put $x = -1$

$f(-1) \Rightarrow 4(-1) - 16 = B(-1-3)$

$-20 = -4B$

$B = \frac{-20}{-4}$

$B = 5$

→ Put $x = 3$

$$4(3) - 16 = A(3+1) + B(3-3)$$
$$-4 = 4A$$
$$A = -1$$

Substitute the values of A and B in the equation

$$\frac{4x-16}{(x-3)(x+1)} = \frac{-1}{x-3} + \frac{5}{(x+1)}$$

$$\int \frac{4x-16}{(x-3)(x+1)} = \int \frac{-1}{(x-3)}\,dx + \int \frac{5}{(x+1)}\,dx$$

Let $u = x - 3$

$$\frac{du}{dx} = 1$$
$$dx = \frac{du}{1}$$

$$\int \frac{-1}{(x-3)}\,dx = \int \frac{-1}{u} \cdot du = -1\int \frac{1}{u}\,du = -1 \ln u = -1 \ln(x-3)$$

Let $u = x + 1$

$$\frac{du}{dx} = 1$$
$$dx = \frac{du}{1}$$

$$\int \frac{5}{x+1}\,dx = \int \frac{5}{u} \cdot du = 5\int \frac{1}{u}\,du = 5 \ln u = 5 \ln(x+1)$$

$$\therefore \int \frac{4x-16}{(x-3)(x+1)} = -1 \ln(x-3) + 5 \ln(x+1)$$

$$\int \frac{4x-16}{(x-3)(x+1)} = -1 \ln(x-3) + 5 \ln(x+1) + C_{//}$$

3) $\int \frac{(2x^2 - 9x - 35)}{(x+1)(x-2)(x+3)} dx$

Solution

Resolving into partial fraction

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} \equiv \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} \equiv \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

Equating the numerator

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

Put $x = 2$

$$f(2) \Rightarrow 2(2)^2 - 9(2) - 35 = B(2+1)(2+3)$$

$$-45 = 15B$$

$$B = \frac{-45}{15}$$

$$B = -3$$

Put $x = -1$

$$f(-1) \Rightarrow 2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3)$$

$$-24 = -6A$$

$$A = \frac{-24}{-6}$$

$$A = 4$$

Put $x = -3$

$$f(-3) \Rightarrow 2(-3)^2 - 9(-3) - 35 = C(-3+1)(-3-2)$$

$$10 = 10C$$

$$C = \frac{10}{10}$$

$$C = 1$$

Substitute the values of A, B and C in the equation.

$$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} + \frac{(-3)}{x-2} + \frac{1}{x+3}$$

$$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \int \frac{4}{x+1}\,dx - \int \frac{3}{x-2}\,dx + \int \frac{1}{x+3}\,dx$$

Let $x+1 = u$

$$\frac{du}{dx} = 1$$

$$dx = \frac{du}{1}$$

$$\int \frac{4}{x+1}\,dx = \int \frac{4}{u}\,.\,du = 4\int \frac{1}{u}\,du = 4\ln u = 4\ln(x+1)$$

Let $x-2 = u$

$$\frac{du}{dx} = 1$$

$$dx = \frac{du}{1}$$

$$\int \frac{3}{x-2}\,dx = \int \frac{3}{u}\,.\,du = 3\int \frac{1}{u}\,du = 3\ln u = 3\ln(x-2)$$

Let $u = x+3$

$$\frac{du}{dx} = 1$$

$$dx = \frac{du}{1}$$

$$\int \frac{1}{x+3}\,dx = \int \frac{1}{u}\,.\,du = \int \frac{1}{u}\,du = \ln u = \ln(x+3)$$

$$\therefore \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3)$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + C$$