NAME: ABIE, UNWANMENE CHIAMAKA
MATRIC NO: 19/MHS01/043 19/MHS01/043
DEPARTMENT: MEDICINE AND SURGERY
COURSE: MAT 104

ASSIGNMENT

1 $\int \frac{11-3x}{x^2+2x-3}\,dx$

SOLUTION

Expressing $\frac{11-3x}{x^2+2x-3}$ as a partial fraction

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$11-3x = A(x+3) + B(x-1)$$

where $x+3=0$, $x=-3$

$$11-3(-3) = A(-3+3) + B(-3-1)$$

$$11+9 = 0-4B$$

$$\frac{4B}{4} = -\frac{20}{4}$$

$$B = -5$$

where $x-1=0$, $x=1$

$$11-3(1) = A(1+3) + B(1-1)$$

$$8 = 4A + 0$$

$$\frac{4A}{4} = \frac{8}{4}$$

$$A = 2$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{2}{x-1} + \left(\frac{5}{x+3}\right)$$

$$\int \frac{11-3x}{(x-1)(x+3)}\,dx$$

$$= \int \frac{2}{x-1}\,dx + \int \frac{-5}{x+3}\,dx$$

Since,

$$\int \frac{n}{ax+b}\,dx = \frac{A\ln(ax+b)}{a} + C$$

$$\therefore \int \frac{11-3x}{(x-1)(x+3)}$$

$$= 2\ln(x-1) - 5\ln(x+3)$$

(2) $\int \frac{4x-16}{x^2-2x-3}\,dx$

Resolving $\frac{4x-16}{x^2-2x-3}$ into partial fractions

$$\frac{4x-16}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$4x-16 = A(x-3) + B(x+1)$$

where $x-3=0$, $x=3$

$$4(3)-16 = A(3-3) + B(3+1)$$

$$-4 = 0 + 4B$$

$$\frac{4B}{4} = \frac{-4}{4}$$

$$B = -1$$

where $x+1=0$, $x=-1$

$$4(-1)-16 = A(-1-3) + B(-1+1)$$

$$-20 = -4A + 0$$

$$\frac{-4A}{4} = \frac{-20}{-4}$$

$$A = 5$$

$$\frac{4x-16}{x^2-2x-3} = \frac{5}{x+1} + \frac{(-1)}{x-3}$$

$$\int \frac{4x-16}{x^2-2x-3}\,dx = \int \frac{5}{x+1}\,dx + \int \frac{-1}{x-3}\,dx$$

$$\int \frac{4x-16}{x^2-2x-3}\,dx = 5\ln(x+1) - \ln(x-3)$$

(3) $\int \frac{(2x^2 - 9x - 35)}{(x+1)(x-2)(x+3)}\,dx$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

When $x + 1 = 0$, $x = -1$

$$2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3) + B(-1+1)(-1+3) + C(-1+1)(-1-2)$$

$$2(1) + 9 - 35 = A(-3)(2) + 0 + 0$$

$$\frac{-24}{-6} = \frac{-6A}{-6}$$

$$A = 4$$

When $x - 2 = 0$, $x = 2$

$$2(2)^2 - 9(2) - 35 = A(2-2)(2+3) + B(2+1)(2+3) + C(2+1)(2-2)$$

$$2(4) - 18 - 35 = 0 + B(3)(5) + 0$$

$$\frac{-45}{15} = \frac{15B}{15}$$

$$B = -3$$

When $x + 3 = 0$, $x = -3$

$$2(-3)^2 - 9(-3) - 35 = A(-3-2)(-3+3) + B(-3+1)(-3+3) + C(-3+1)(-3-2)$$

$$2(9) + 27 - 35 = 0 + 0 + C(-2)(-5)$$

$$\frac{10}{10} = \frac{10}{10}C$$

$$C = 1$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} + \frac{(-3)}{x-2} + \frac{1}{x+3}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}\,dx = \int \frac{4}{x+1}\,dx + \int \frac{-3}{x-2}\,dx + \int \frac{1}{x+3}\,dx$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3)$$