

MAT NO: 19/MHS01/045

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COURSE: MAT 104

$$\int \frac{11-3x}{x^2+2x-3}$$

$$\text{Solution} = \frac{11-3x}{(x^2+3x)(x-3)} = \frac{11-3x}{(x+3)(x-1)(x+3)} = \frac{11-3x}{(x-1)(x+3)}$$

$$\frac{A}{x-1} + \frac{B}{x+3} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

multiply all by $(x-1)(x+3)$

$$A(x+3) + B(x-1) = 11-3x \quad \text{At}$$

At $x=3$, we have

$$B(-3-1) = 11-3(-3)$$

$$B(-4) = 20$$

$$\frac{-4B}{-4} = \frac{20}{-4}$$

$$B = -5$$

At

At $x=1$, we have

$$A(x+3) = 11-3x$$

$$A(4) = 11-3(1)$$

$$4A = 8$$

$$A = \frac{8}{4} = 2$$

$$\int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx = \int \frac{11-3x}{x^2+2x-3} dx$$

$$\Rightarrow \int \frac{2dx}{x-1} + \int \frac{-5dx}{x+3} = \int \frac{11-3x}{x^2+2x-3} dx$$

$$\text{Let } u = x-1$$

$$\frac{du}{dx} = 1$$

$$dx = du$$

$$\Rightarrow 2 \int \frac{du}{u}$$

$$2 \ln u$$

$$2 \ln(x-1) - 5 \ln(x+3)$$

$$u = x+3$$

$$\frac{du}{dx} = 1$$

$$dx = du$$

$$\Rightarrow -5 \int \frac{du}{u}$$

$$-5 \ln u$$

$$2) \int \frac{4x-16}{x^2-2x-3}$$

Solution

$$\frac{4x-16}{x^2-2x-3} = \frac{4x^2-16}{(x^2-3x)+(x-3)} = \frac{4x^2-16}{(x+1)(x-3)}$$

$$= \frac{A}{x+1} + \frac{B}{x-3} = \frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$$

Multiply all by $(x+1)(x-3)$

$$A(x-3) + B(x+1) = 4x^2-16$$

At $x=3$, we have.

$$B(x+1) = 4x-16$$

$$B(3+1) = 4(3)-16$$

$$4B = 12-16$$

$$4B = -4$$

$$B = -1$$

$$\int \frac{5}{x+1} + \int \frac{-1}{x-3} dx = \int \frac{4x-16}{x^2-2x-3} dx$$

$$\text{let } u = x+1$$

$$du = dx$$

$$dx = du$$

$$\Rightarrow 5 \int \frac{du}{u}$$

$$= 5 \ln |u|$$

$$\Rightarrow 5 \ln(x+1) - \ln(x-3)$$

At $x = -1$

$$A(x-3) = 4x-16$$

$$A(-1-3) = 4(-1)-16$$

$$-4A = -4-16$$

$$-4A = -20$$

$$A = 5$$

$$u = x-3$$

$$du = dx$$

$$dx = du$$

$$= -1 \int \frac{du}{u}$$

$$= -1 \ln |u|$$

$$3) \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx$$

$$= \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$$

$$= \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$2x^2 - 9x - 35 = A(x^2 + 3x - 2x - 6) + B(x^2 + 3x + x + 3) + C(x^2 - 2x + x - 2)$$

$$2x^2 - 9x - 35 = A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2)$$

$$2x^2 - 9x - 35 = Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$2x^2 - 9x - 35 = x^2(A+B+C) + x(A+4B-C) + (-6A+3B-2C)$$

$$A+B+C = 2 \quad \text{--- (1)}$$

$$A+4B-C = -9 \quad \text{--- (2)}$$

$$-6A+3B-2C = -35 \quad \text{--- (3)}$$

$$\text{From eqn (1), } A = 2 - B - C \quad \text{--- (4)}$$

Put eqn (4) in eqn (2) & (3)

$$*(2 - B - C) + 4B - C = -9 \Rightarrow 2 + 3B - C = -9 \rightarrow 3B - C = -11$$

$$*-6(2 - B - C) + 3B - 2C = -35 \Rightarrow -12 + 6B + 6C + 3B - 2C = -35$$

$$\Rightarrow -12 + 9B + 4C = -35 \Rightarrow 9B + 4C = -23$$

$$3B - 2C = -11 \quad \times 4$$

$$9B - 4C = -44 \quad \times 2$$

$$12B - 8C = -44$$

$$18B - 8C = -46$$

$$30B = -90$$

$$\frac{30}{30} = \frac{-90}{30}$$

$$B = -3$$

$$\therefore 3B - 2C = -11$$

$$3(-3) - 2C = -11$$

$$-9 - 2C = -11$$

$$2C = -11 + 9$$

$$\frac{-2C}{2} = \frac{-2}{2}$$

$$C = 1$$

$$A = 2 - B - C$$

$$= 2 - (-3) - 1 = 4$$

$$A = 4$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} + \frac{-3}{x-2} + \frac{1}{x+3}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} dx + \int \frac{-3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$= 4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3| + C$$