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19/MHS01/304

MBBS

MTH 109

1. $\int \frac{11-3x}{x^2+2x-3}$

$$\frac{11-3x}{x^2+2x-3} = \frac{-11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

$$A(x+3) + B(x-1) = 11-3x$$

At $x = -3$

$$B(-4) = 11 - 3(-3) = 11 + 9$$

$$B = \frac{20}{-4} = -5$$

At $x = 1$, we have

$$A(4) = 11 - 3$$

$$A(4) = 8$$

$$A = 2$$

Therefore

$$\int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx = \int \frac{11-3x}{x^2+2x-3} dx \quad \text{pt}$$

$$= 2 \ln(x-1) - 5 \ln(x+3)$$

$$\int \frac{4x-16}{x^2-2x-3} dx = \frac{4x-16}{(x+1)(x-3)}$$

$$= \frac{A}{(x+1)} + \frac{B}{(x-3)}$$

$$A(x-3) + B(x+1) = 4x-16$$

$$\text{At } x=3$$

$$B(4) = 12-16$$

$$B(4) = -4$$

$$B = -1$$

$$\text{At } x=-1$$

$$A(-4) = -4-16$$

$$A(-4) = -20$$

$$A = 5$$

Therefore

$$\int \frac{5}{(x+1)} dx + \int \frac{-1}{(x-3)} dx = \int \frac{4x-16}{x^2-2x-3} dx$$

$$= 5 \ln |x+1| - \ln |x-3| + C$$

$$3 \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2) = 2x^2 - 9x - 35$$

$$Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C = 2x^2 - 9x - 35$$

$$x^2(A+B+C) + x(A+4B-C) - 6A+3B-2C = 2x^2 - 9x - 35$$

$$A + B + C = 2 \quad \text{--- (i)}$$

$$A + 4B - C = -9 \quad \text{--- (ii)}$$

$$-6A + 3B - 2C = -35 \quad \text{--- (iii)}$$

$$A = 2 - B - C \quad \text{--- (iv)}$$

Substitute eqn (iv) in eqn (ii)

$$2 - B - C + 4B - C = -9$$

$$-2C + 3B = -11 \quad \text{--- (v)}$$

Substitute eqn (v) into eqn (iii)

$$-6(2 - B - C) + 3B - 2C = -35$$

$$-12 + 6B + 6C + 3B - 2C = -35$$

$$9B + 4C = -23 \quad \text{--- (vi)}$$

$$-2C + 3B = -11 \quad \times 3$$

$$4C + 9B = -23 \quad \times 1$$

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$$-6C + 9B = -33$$

$$4C + 9B = -23$$

$$\frac{+10C - 10C}{-10C} = \frac{+10}{-10}$$

$$\frac{+10}{-10} = \frac{+10}{-10}$$

$$C = 1$$

Using eq (vi)

$$4C + 9B = -23$$

$$4C + 9B = -23$$

$$4 + 9B = -23$$

$$9B = -23 - 4$$

$$\frac{9B}{9} = \frac{-27}{9}$$

$$B = -3$$

Using eq (w)

$$A = 2 - B - C$$

$$A = 2 - (-3) - 1$$

$$A = 5 - 1$$

$$A = 4$$

Therefore

$$\frac{4}{x+1} + \frac{-3}{x-2} + \frac{1}{x+3}$$

$$= \int \frac{4}{x+1} dx + \int \frac{-3}{x-2} dx + \int \frac{1}{x+3} dx = \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}$$

$$\therefore 4 \ln |x+1| - 3 \ln |x-2| + \ln |x+3|$$