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DEPT: MEDICINE AND SURGERY
MATRIC NUMBER: 19/MHS01/177

MAT 104 ASSIGNMENT

1. $\int 11-3x/(x^2+2x-3)\,dx$

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

Multiplying both sides by $(x-1)(x+3)$

$11-3x = A(x+3)+B(x-1)$

At $x = 1$

$11-3(1) = A(1+3)+B(1-1)$

$11-3 = 4A$

$4A = 8$

$A = \frac{8}{4} \Rightarrow 2$

At $x = -3$

$11-3(-3) = A(-3+3)+B(-3-1)$

$11+9 = -4B$

$-4B = 20$

$B = \frac{20}{-4} \Rightarrow -5$

Hence $\frac{11-3x}{(x-1)(x+3)} = \frac{2}{x-1} - \frac{5}{x+3}$

$$\int \frac{11-3x}{(x^2+2x-3)}\,dx = \int \frac{2}{x-1}\,dx - \int \frac{5}{x+3}$$

for $\int \frac{2}{x-1}\,dx$

let $x-1$ be u

$du = 1\,dx$

$dx = du$

$$\int \frac{2}{x-1}\,dx = \int \frac{2}{u}\,du \Rightarrow 2\int \frac{du}{u} \Rightarrow 2\ln u$$

For $-\int \frac{5}{x+3}\,dx$

Let $x+3$ be t

$dt = 1\,dx$

$dx = dt$

$$-\int \frac{5}{x+3}\,dx = -5\int \frac{dt}{t} = -5\ln t$$

$$\int \frac{11-3x}{(x^2+2x-3)} = 2\ln u - 5\ln t \quad - (1)$$

substituting the values of u and t into (1)

$$\int \frac{11-3x}{x^2+2x-3} = 2\ln(x-1) - 5\ln(x+3)$$

2. $\int \frac{4x-16}{x^2+2x-3}\, dx$

$\frac{4x-16}{x^2-2x-3} = \frac{4x-16}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)}$

$\frac{4x-16}{(x+1)(x-3)} = \frac{A(x-3)+B(x+1)}{(x+1)(x+3)}$

Multiplying both sides by $(x+1)(x-3)$

$4x-16 = A(x-3)+B(x+1)$

At $x=-1$

$\cancel{4(-1)}\ 4(-1)-16 = A(-1-3)+B(-1+1)$

$-4-16 = -4A$

$-4A = -20$

$A = \frac{-20}{-4} => 5$

At $x=3$

$4(3)-16 = A(3-3)+B(3+1)$

$12-16 = 4B$

$4B = -4$

$B = \frac{-4}{4} => -1$

Hence $\int \frac{4x-16}{x^2-2x-3}\, dx = \int \frac{5}{x+1}\, dx - \int \frac{dx}{x-3}$

For $\int \frac{5}{x+1}\, dx$ let $x+1$ be u

$du = 1\, dx \therefore dx = du$

$\int \frac{5}{x+1}\, dx = 5\int \frac{du}{u} => 5 \ln u$

For $-\int \frac{dx}{x-3} \cancel{dx} = \cancel{-1} \int du$ let $x-3$ be t

$dt = 1\, dx \therefore dx = dt$

$-\int \frac{d\cancel{x}}{x-3} = -1\int \frac{dt}{t} => -\ln t$

$$\int \frac{4x-16}{x^2-2x-3} = 5\ln v - \ln t \quad - ⊛$$

Substituting the values of v and t into ⊛

$$\int \frac{4x-16}{x^2-2x-3} = 5\ln(x+1) - \ln(x-3)$$

3. $$\int \frac{(2x^2-9x-35)}{(x+1)(x-2)(x+3)}\,dx$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

Multiplying both sides by $(x+1)(x-2)(x+3)$

$2x^2-9x-35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$

At $x = -1$

$2(-1)^2-9(-1)-35 = A(-1-2)(-1+3) + B(-1+1)(-1+3) + C(-1+1)(-1-2)$

$2+9-35 = A(-3)(2)$

$11-35 = -6A$

$-6A = -24$

$A = \frac{-24}{-6} \Rightarrow 4$

At $x = 2$

$2(2)^2-9(2)-35 = A(2-2)(2+3) + B(2+1)(2+3) + C(2+1)(2-2)$

$8-18-35 = B(3)(5)$

$-10-35 = 15B$

$15B = -45$

$B = \frac{-45}{15} \Rightarrow -3$

At $x = -3$

$2(-3)^2-9(-3)-35 = A(-3-2)(-3+3) + B(-3+1)(-3+3) + C(-3+1)(-3-2)$

$18+27-35 = C(-2)(-5)$

$45-35 = 10C$

$10C = 10$

$e = \frac{10}{10} => 1$

Hence $\int \frac{(2x^2-9x-35)}{(x+1)(x+2)(x+3)} = \int \frac{4}{x+1}dx - \int \frac{3}{x-2}dx + \int \frac{dx}{x+3}$

For $\int \frac{4}{x+1}dx$ let $x+1$ be u

$du = 1\,dx \therefore dx = du$

$\int \frac{4}{x+1}dx = 4\int \frac{du}{u} => 4\ln u$

For $-\int \frac{3}{x-2}dx$ let $x-2$ be t

$dt = 1\,dx \therefore dx = dt$

$-\int \frac{3}{x-2}dx = -3\int \frac{dt}{t} => -3\ln t$

For $\int \frac{dx}{x+3}$ let $x+3$ be v

$dv = 1\,dx \therefore dx = dv$

$\int \frac{dx}{x+3} = +\int \frac{dv}{v} => \ln v$

$\therefore \int \frac{(2x^2-9x-35)}{(x+1)(x-2)(x+3)} = 4\ln u - 3\ln t + \ln v$ ⊛

substituting the values of u, t and v into ⊛

$\int \frac{(2x^2-9x-35)}{(x+1)(x-2)(x+3)} = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3)$