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1. $\int \frac{11-3x}{x^2+2x-3} dx$

From the denominator: $x^2+2x-3=0$

factors are $\Rightarrow +3$ and -1

$$x^2+2x-3=0$$

$$x^2+3x-x-3=0$$

$$(x^2+3x)(-x-3)=0$$

$$x(x+3)-1(x+3)=0$$

$$(x+3)(x-1)=0 \quad ; \quad x=-3 \text{ and } 1$$

$$\therefore \int \frac{11-3x}{x^2+2x-3} dx = \int \frac{11-3x}{(x+3)(x-1)} dx$$

Resolve $\int \frac{11-3x}{(x+3)(x-1)}$ into partial fraction

$$\frac{11-3x}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$$

$$\frac{11-3x}{(x+3)(x-1)} = \frac{A(x-1)}{x+3} + \frac{B(x+3)}{x-1}$$

(2)

Equating the numerators

$$11 - 3x = A(x-1) + B(x+3) \dots \textcircled{1}$$

Put $x=1$ into the equation $\textcircled{1}$

$$11 - 3(1) = A(1-1) + B(1+3)$$

$$8 = 0 + 4B$$

$$\frac{8}{4} = \frac{4B}{4}$$

$$B = 2$$

Put $x=-3$ into the equation $\textcircled{1}$

$$11 - 3(-3) = A(-3-1) + B(-3+3)$$

$$20 = -4A + 0$$

$$\frac{20}{-4} = \frac{-4A}{-4}$$

$$A = -5$$

$$\therefore \int \frac{11-3x}{x^2+2x-3} dx = \int \frac{-5}{(x+3)} + \frac{2}{(x-1)}$$

$$\Rightarrow \int \frac{-5}{(x+3)} dx + \int \frac{2}{(x-1)} dx$$

$$\Rightarrow -5 \int \frac{1}{(x+3)} dx + 2 \int \frac{1}{(x-1)} dx$$

$$\Rightarrow -5 \ln(x+3) + 2 \ln(x-1) + c$$

(3)

$$2. \int \frac{4x-16}{(x^2-2x-3)} dx$$

from the denominator: $x^2 - 2x - 3$

factors are $\Rightarrow -3$ and $+1$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$(x^2 - 3x) + (x - 3) = 0$$

$$x(x-3) + 1(x-3) = 0$$

$$(x-3)(x+1) = 0 \quad ; x = 3 \text{ and } -1$$

$$\int \frac{4x-16}{x^2-2x-3} dx = \int \frac{4x-16}{(x-3)(x+1)} dx$$

Resolve $\int \frac{4x-16}{(x-3)(x+1)}$ into partial fraction.

$$\frac{4x-16}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)}$$

$$\frac{4x-16}{(x-3)(x+1)} = \frac{A(x+1)}{(x-3)} + \frac{B(x-3)}{(x+1)}$$

Equating the numerals.

$$4x - 16 = A(x+1) + B(x-3) \quad \dots \textcircled{i}$$

Put $x = 3$ into the equation \textcircled{i}

$$4(3) - 16 = A(3+1) + B(3-3)$$

$$-4 = 4A + 0$$

(4)

$$\frac{-4}{4} = \frac{4A}{4}$$

$$A = -1$$

Put $x = -1$ into equation (i)

$$4(-1) - 16 = A(-1+1) + B(-1-3)$$

$$-20 = 0 + 4B$$

$$\frac{-20}{-4} = \frac{4B}{-4}$$

$$5 = -B$$

$$B = -5$$

$$\therefore \int \frac{4x-16}{x^2-2x-3} dx = \int \frac{-1}{x-3} + \frac{5}{x+1} dx$$

$$\Rightarrow \int \frac{-1}{x-3} + \int \frac{5}{x+1} dx = -1 \int \frac{1}{x-3} dx + 5 \int \frac{1}{x+1} dx$$

$$\Rightarrow -\ln|x-3| + 5\ln|x+1| + c$$

$$3: \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx$$

Resolve $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} dx$ into partial fraction.

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$x = -1 ; x = 2 ; x = -3$$

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$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

Equate the numerators

$$2x^2 - 9x - 35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

Put $x = -1$ into the equation.

$$2(-1)^2 - 9(-1) - 35 = A(-1-2)(-1+3) + B(-1+1)(-1+3) + C(-1+1)(-1+2)$$

$$-24 = -6A + 0 + 0$$

$$\frac{-24}{-6} = \frac{-6A}{-6}$$

$$-6 = -6$$

$$A = 4$$

Put $x = 2$ into the equation

$$2(2)^2 - 9(2) - 35 = A(2-2)(2+3) + B(2+1)(2+3) + C(2+1)(2-2)$$

$$-45 = 0 + 15B + 0$$

$$\frac{-45}{15} = \frac{15B}{15}$$

$$-3 = B$$

$$B = -3$$

Put $x = -3$ into the equation.

$$2(-3)^2 - 9(-3) - 35 = A(-3-2)(-3+3) + B(-3+1)(-3+3) + C(-3+1)(-3-2)$$

$$10 = 0 + 0 + 10C$$

$$\frac{10}{10} = \frac{10C}{10}$$

$$1 = C$$

$$C = 1$$

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$$\therefore \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} + \frac{-3}{x-2} + \frac{1}{x+3} dx$$

$$\Rightarrow \int \frac{4}{x+1} + \frac{-3}{x-2} + \frac{1}{x+3} dx = 4 \int \frac{1}{x+1} - 3 \int \frac{1}{x-2} + \int \frac{1}{x+3} dx$$

$$\Rightarrow 4 \ln|x+1| - 3 \ln|x-2| + \ln|x+3| + C$$