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Assignment: points of intersection of circles & straight lines

Answers

Find the point of intersection of the following line on the circle

1. $x - y - 14 = 0$ and $x^2 + y^2 - 6x + 8y = 0$

$$x - y - 14 = 0 \dots \text{equation (i)}$$

$$x^2 + y^2 - 6x + 8y = 0 \dots \text{equation (ii)}$$

$$y = x - 14 \text{ (insert into equation (ii))}$$

$$x^2 + (x - 14)^2 - 6x + 8(x - 14) = 0$$

$$x^2 + (x - 14)(x - 14) - 6x + 8x - 112 = 0$$

$$x^2 + x^2 - 14x - 14x + 196 - 6x + 8x - 112 = 0$$

$$2x^2 - 28x - 6x + 8x + 196 - 112 = 0$$

$$2x^2 - 26x + 84 = 0$$

Divide through by 2

$$x^2 - 13x + 42 = 0$$

$$x^2 - 7x - 6x + 42 = 0$$

$$x(x - 7) - 6(x - 7) = 0$$

$$(x - 7)(x - 6) = 0$$

$$x = 7 \text{ or } 6$$

when $x = 7$

$$y = 7 - 14$$

$$y = -7$$

when $x = 6$

$$y = 6 - 14$$

$$y = -8$$

\therefore The point of intersection = (7, -7) and (6, -8)

$$2. \quad 2x + y - 10 = 0 \quad \text{and} \quad x^2 + y^2 + 4x - 6y = 0$$

$$y = -2x + 10 \quad \dots \text{equation (i)}$$

$$x^2 + y^2 + 4x - 6y = 0 \quad \text{equation (ii)}$$

Insert equation (i) into (ii)

$$x^2 + (-2x + 10)^2 + 4x - 6(-2x + 10) = 0$$

$$x^2 + (-2x + 10)(-2x + 10) + 4x + 12x - 60 = 0$$

$$x^2 + 4x^2 - 20x - 20x + 100 + 4x + 12x - 60 = 0$$

$$5x^2 + 4x^2 - 40x + 4x + 12x + 100 - 60 = 0$$

$$5x^2 - 24x + 40 = 0$$

Using quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \text{Complex roots}$

\therefore The line and the circle do not intersect

$$3. \quad x - 5y - 2 = 0 \quad \text{and} \quad x^2 + 25y^2 - 6xy - 16 = 0$$

$$x = 5y + 2 \quad \dots \dots \text{equation (i)}$$

$$x^2 + 25y^2 - 6xy - 16 = 0 \quad \dots \dots \text{equation (ii)}$$

insert equation (i) into equation (ii)

$$(5y + 2)^2 + 25y^2 - 6y(5y + 2) - 16 = 0$$

$$(5y + 2)(5y + 2) + 25y^2 - 6y(5y + 2) - 16 = 0$$

$$25y^2 + 10y + 10y + 4 + 25y^2 - 30y^2 - 12y - 16 = 0$$

$$50y^2 + 20y - 30y^2 + 4 - 16 = 0$$

$$50y^2 - 30y^2 + 20y - 12y + 4 - 16 = 0$$

$$20y^2 + 8y - 12 = 0$$

Divide through by 2

$$10y^2 + 4y - 6 = 0$$

$$10y^2 + 10y - 6y - 6 = 0$$

$$10y(y+1) - 6(y+1) = 0$$

$$(y+1)(10y-6) = 0$$

$$10y - 6 = 0$$

$$y = -1 \text{ or } 6/10$$

$$y = -1 \text{ or } 3/5$$

when $y = -1$

$$x = 5(-1) + 2 = -3$$

when $y = 3/5$

$$x = 5(3/5) + 2 = 5$$

The point of intersection are

$(-3, -1)$ and $(5, 3/5)$

$\therefore (-3, -1) \text{ \& } (5, 3/5)$