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Solution

1. $\int \frac{11-3x}{x^2+2x-3}\,dx$

$\int \frac{11-3x}{(x+3)(x-1)} \Rightarrow \frac{A}{x+3} + \frac{B}{x-1} = \frac{11-3x}{(x+3)(x-1)}$

$\frac{A(x-1)+B(x+3)}{(x+3)(x-1)} = \frac{11-3x}{(x+3)(x-1)}$

multiply both sides by $(x+3)(x-1)$

$A(x-1)+B(x+3) = 11-3x$

$(A+B)x + (3B-A) = 11-3x$

$B + A = 11 \times 3$

$3B - A = -3 \times 1$

$3B + 3A = 33$

$-3B - A = -3$

$4A = 36 \Rightarrow A = 9$

$B + 9 = 11 \Rightarrow B = 2$

$\therefore \int \frac{9\,dx}{x+3} + \int \frac{2\,dx}{x-1} = \int \frac{11-3x}{x^2+2x-3}$

let $v = x+3$ $u = x-1$

$dv = 1dx$ $du = 1dx$

$dx = dv$ $dx = du$

$\int \frac{9\,dv}{v}$ $\int \frac{2\,du}{u}$

$9 \ln v$ $2 \ln u$

$= 9 \ln(x+3) + 2 \ln(x-1)$

2. $\int \frac{4x-16}{x^2-2x-3}\,dx$

$\int \frac{4x-16}{(x+1)(x-3)} \Rightarrow \frac{A}{x+1} + \frac{B}{x-3} = \frac{4x-16}{(x+1)(x-3)}$

Aliyu Mariam Omotayo 19/MHS01/085 028

$$\frac{A(x-3)+B(x+1)}{(x+1)(x-3)} = \frac{4x-16}{(x+1)(x-3)}$$

multiply both sides by $(x+1)(x-3)$

$A(x-3)+B(x+1) = 4x-16 \Rightarrow (A+B)x + (B-3A)$

$A+B = 4 \times 3$

$-3A+B = -16 \times 1$

$4B = -4 \Rightarrow B = -1$

$A+(-1) = 4 \Rightarrow A = 5$

$\therefore \int \frac{5\,dx}{x+1} + \int \frac{-1\,dx}{x-3} = \int \frac{4x-16}{x^2-2x-3}$

let $u = x+1$ $\quad u = x-3$

$du = 1dx$ $\quad du = 1dx$

$dx = du$ $\quad dx = du$

$\int \frac{5du}{u}$ $\quad \int \frac{-1du}{u}$

$5\ln u$ $\quad -1\ln u$

$\therefore = 5\ln(x+1) - 1\ln(x-3)$

3. $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}dx$

$\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3} = \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$

$\frac{A(x-2)(x+3)+B(x+1)(x+3)+C(x+1)(x-2)}{(x+1)(x-2)(x+3)} = \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$

$\frac{A(x^2+x-6)+B(x^2+4x+3)+C(x^2-x+2)}{(x+1)(x-2)(x+3)} = \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$

multiply both sides by $(x+1)(x-2)(x+3)$

$A(x^2+x-6)+B(x^2+4x+3)+C(x^2-x-2) = 2x^2-9x-35$

$Ax^2+Ax-6A+Bx^2+4Bx+3B+Cx^2-Cx-2C = 2x^2-9x-35$

$(A+B+C)x^2+(A+4B-C)x-6A+3B-2C = 2x^2-9x-35$

$A+B+C = 2$ ——— (i)

$A+4B-C = -9$ ——— (ii)

$-6A+3B-2C = -35$ ——— (iii)

from (i) make B subject of the formular, $B = 2-A-C$

Aliyu Mariam Omotayo 19/MHS01/085 028

Then substitute for B in equ (ii) and (iii)

$A + 4(2 - A - C) - C = -9$

$-3A - 5C = -17$ ——— (iv)

$-6A + 3(2 - A - C) - 2C = -35$

$-9A - 5C = -41$ ——— (v)

Subtract equ (iv) from equ (v)

$-9A - 5C = -41$

$-3A - 5C = -17$

$-6A = -24 \implies A = 4$

$-9(4) - 5C = -41 \implies C = -5$

Substitute A and C in equ (i)

$4 + B + (-5) = 2$

$B + 4 - 5 = 2$

$B = 3$

$$\therefore \int \frac{4\,dx}{x+1} + \int \frac{3\,dx}{x-2} + \int \frac{-5\,dx}{x+3} = \int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}$$

let $u = x + 1$ $\quad u = x - 2$ $\quad u = x + 3$

$du = 1\,dx$ $\quad du = 1\,dx$ $\quad u = 1\,dx$

$dx = du$ $\quad dx = du$ $\quad dx = du$

$\int \frac{4\,du}{u}$ $\quad \int \frac{3\,du}{u}$ $\quad \int \frac{-5\,du}{u}$

$4 \ln u$ $\quad 3 \ln u$ $\quad -5 \ln u$

$\therefore = 4 \ln(x+1) + 3 \ln(x-2) - 5 \ln(x+3)$