$= 11+9 = A(3-3) + B(9$

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MAT 104

1) $\int \frac{11-3x}{(x^2+2x-3)}\,dx$ 2) $\int \frac{4x-16}{(x^2-2x-3)}\,dx$ 3) $\int \frac{2x^2-9x-35}{(x+1)(x+3)}$

Solution

$$\int \frac{11-3x}{(x^2+2x-3)}\,dx = \int \frac{11-3x}{(x+3)(x-1)}\,dx$$

$$\frac{11-3x}{(x+3)(x-1)} = \frac{A}{(x+3)} + \frac{B}{(x-1)}$$

$$\frac{11-3x}{(x+3)(x-1)} = \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}$$

$$11-3x = A(x-1)+B(x+3)$$

$$f(1) = 11-3(1) = A(1-1)+B(1+3)$$

$$= 11-3 = A(0)+B(4)$$

$$\frac{8}{4} = \frac{4B}{4}$$

$$\therefore B = 2$$

For A,

$$f(-3) = 11-3(-3) = A(-3-1)+B(-3+3)$$

$$\Rightarrow 11+9 = A(-4)+2(0)$$

$$\Rightarrow \frac{20}{-4} = \frac{-4A}{-4}$$

$$\therefore A = +5$$

$$\int \frac{11-3x}{(x+3)(x-1)}\,dx = \int \frac{-5}{(x+3)}\,dx + \int \frac{2}{(x-1)}\,dx$$

Let $u = x+3$

$\frac{du}{dx} = 1$

$\therefore dx = du$

$\Rightarrow \int \frac{-5}{(x+3)} dx = \int \frac{-5}{u} du$

$-5\int \frac{1}{u} du$

$\therefore -5\ln u + c$

$\Rightarrow -5\ln(x+3) + c$ //

Let $u = x-1$

$\frac{du}{dx} = 1$

$\therefore dx = du$

$\int \frac{2}{u} dx = \int \frac{2}{u} du$

$2\int \frac{1}{u} du$

$2\ln u + c$

$2\ln(x-1) + c$ //

$\therefore \int \frac{11-3x}{(x+3)(x-1)} dx = -5\ln(x+3) + 2\ln(x-1) + c$ //

2 $\int \frac{4x-16}{(x^2-2x-3)} dx = \int \frac{4x-16}{(x-3)(x+1)} dx$

$\frac{4x-16}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)}$

$\frac{4x-16}{(x-3)(x+1)} = \frac{A(x+1) + B(x-3)}{(x-3)(x+1)}$ —

$4x-16 = A(x+1) + B(x-3)$

$f(3) = 4(3) - 16 = A(3+1) + B(3-3)$

$12 - 16 = A(4) + B(0)$

$\frac{-4}{4} = \frac{4A}{4}$

$\therefore A = -1$ //

for B,

$f(-1) = 4(-1) - 16 = A(-1+1) + B(-1-3)$

$= -4 - 16 = -1(0) + B(-4)$

$\frac{-20}{-4} = \frac{-4B}{-4}$

$5 = B$

$\therefore B = 5$

$$\int \frac{4x-16}{(x-3)(x+1)}\,dx = \int \frac{-1}{(x-3)}\,dx + \int \frac{5}{(x+1)}\,dx$$

let $x - 3 = u$

$\frac{du}{dx} = 1$

$\therefore dx = du$

$\int \frac{-1}{u}\,du$

$-1 \int \frac{1}{u}\,dx$

$-1 \ln u + C$

$= -1 \ln(x-3) + C$ //

let $x+1 = u$

$\frac{du}{dx} = 1$

$dx = du$

$\int \frac{5}{u}\,du$

$5 \int \frac{1}{u}\,du$

$5 \ln u + C \quad = 5 \ln(x+1) + C$ //

$$\int \frac{9x-16}{(x-3)(x+1)}\,dx = -\ln(x-3) + 5\ln(x+1) + C //$$

3 $$\int \frac{2x^2-9x-35}{(x+1)(x+3)}\,dx = \frac{A}{(x+1)} + \frac{B}{(x+3)}$$

$$\frac{2x^2-9x-35}{(x+1)(x+3)} = \frac{A(x+3)+B(x+1)}{(x+1)(x+3)}$$

$$2x^2-9x-35 = A(x+3)+B(x+1)$$

$$f(-3) = 2(-3)^2-9(-3)-35 = A(-3+3)+B(-3+1)$$

$$= 2(9)+27-35 = A(0)+B(-2)$$

$$18-8 = -2B$$

$$\frac{10}{-2} = \frac{-2B}{-2}$$

$$\therefore B = -5 //$$

For A,

~~Let~~ $x = -1$

$$f(-1) = 2(-1)^2-9(-1)-35 = A(-1+3)+B(-1+1)$$

$$= 2+9-35 = A(2)+B(0)$$

$$11-35 = 2A$$

$$\frac{24}{2} = \frac{2A}{2}$$

$$\therefore A = 12$$

$$\int \frac{2x^2-9x-35}{(x+1)(x+3)}\,dx = \int \frac{12}{(x+1)}\,dx + \int \frac{-5}{(x+3)}$$

Let $u = x+1$

$$\frac{du}{dx} = 1$$

$$\therefore dx = du$$

$$\int \frac{12}{u}\,du$$

$$12\int \frac{1}{u}\,du$$

$\rightarrow 12 \ln u + c$

$= 12 \ln(x+1) + c$

Now let $u = x+3$

$$\frac{du}{dx} = 1$$

$$dx = du$$

$$\therefore \int \frac{-5}{u}\, du$$

$$-5 \int \frac{1}{u}\, du$$

$$= -5 \ln u + c$$

$$= -5 \ln(x+3) + c$$

$$\therefore \int \frac{2x^2 - 9x - 35}{(x+1)(x+3)}\, dx = 12 \ln(x+1) - 5 \ln(x+3) + c$$