

MAI Assignment

1

$$\int \frac{11-3x}{(x^2+2x-3)} dx$$

Solution

$$\begin{array}{l} x^2+2x-3 \\ x^2+3x-x-3 \\ x(x+3)-1(x+3) \\ (x-1)(x+3) \end{array} \quad \left| \quad \begin{array}{l} \frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)} \\ = \frac{A}{x-1} + \frac{B}{x+3} \Rightarrow \frac{A(x+3) + B(x-1)}{(x-1)(x+3)} \end{array} \right.$$

$$A(x+3) + B(x-1) = 11-3x$$

$$Ax + 3A + Bx - B = 11-3x$$

$$(A+B)x + (3A-B) = 11-3x$$

(Comparing) $A+B = -3 \dots \textcircled{1}$

$$3A-B = 11 \dots \textcircled{2}$$

$$4A = 8 \Rightarrow A = 2$$

$$2 + B = -3 \quad (\text{Put in equation 1})$$

$$B = -3 - 2 \Rightarrow B = -5$$

We can now write

$$\int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx = \int \frac{11-3x}{x^2+2x-3}$$

$$\text{let } u = x-1$$

$$u = x+3$$

$$du = dx$$

$$du = dx$$

$$\Rightarrow \int 2 du$$

$$\Rightarrow \int -5 du$$

We can now write

$$\int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx = \int \frac{11-3x}{x^2+2x-3}$$

$$\text{let } u = x-1$$

$$u = x+3$$

$$du = dx$$

$$du = dx$$

$$\Rightarrow \int \frac{2 du}{u}$$

$$\Rightarrow \int \frac{-5 du}{x+3u}$$

$$= 2 \ln u$$

$$\Rightarrow -5 \ln u$$

$$\Rightarrow 2 \ln(x-1) - 5 \ln(x+3) + C$$

2

$$\int \frac{4x-16}{x^2-2x-3} dx$$

Solution

$$\begin{array}{l|l} x^2-2x-3 & \int \frac{4x-16}{x^2-2x-3} dx \Rightarrow \int \frac{4x-16}{(x+1)(x-3)} \\ x^2-3x+x-3 & \\ x(x-3)+1(x-3) & = \frac{A}{x+1} + \frac{B}{x-3} \Rightarrow \frac{A(x-3)+B(x+1)}{(x+1)(x-3)} \\ (x+1)(x-3) & \end{array}$$

$$A(x-3) + B(x+1) = 4x-16$$

$$Ax - 3A + Bx + B = 4x - 16$$

$$(A+B)x + (B-3A) = 4x - 16$$

(Comparing sides) $B+A = 4 \dots \textcircled{1}$

$$B-3A = -16 \dots \textcircled{2}$$

$$4A = 20 \Rightarrow A = 5$$

$$4, \quad 4, \quad \text{(Put in equation (1))}$$

$$B+5=4$$

$$B=4-5 \Rightarrow B=-1$$

We can now write

$$\int \frac{5}{x+1} dx + \int \frac{-1}{x-3} dx = \int \frac{4x-16}{x^2-2x-3} dx$$

$$\text{let } u = x+1$$

$$u = x-3$$

$$du = dx$$

$$du = dx$$

$$\Rightarrow \int \frac{5 du}{u}$$

$$\Rightarrow \int \frac{-du}{u}$$

$$\Rightarrow 5 \ln U$$

$$\Rightarrow -\ln U$$

$$\Rightarrow 5 \ln(x+1) - \ln(x-3) + C$$

$$\int \frac{(2x^2 - 9x - 35) dx}{(x+1)(x-2)(x+3)}$$

Solution

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$2x^2 - 9x - 35 = A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2)$$

$$2x^2 - 9x - 35 = Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$2x^2 - 9x - 35 = Ax^2 + Bx^2 + Cx^2 + Ax + 4Bx - Cx - 6A + 3B - 2C$$

$$2x^2 - 9x - 35 = (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C)$$

(Comparing) $A+B+C = 2 \dots \textcircled{1}$

$$A+4B-C = -9 \dots \textcircled{2}$$

$$-6A+3B-2C = -35 \dots \textcircled{3}$$

$$-A+B+C = 2$$

$$A+4B-C = -9 \dots \textcircled{4}$$

$$-3B+2C = 11, \therefore 3B-2C = -11 \text{ (Rt in equation } \textcircled{3})$$

$$\therefore -6A(-11) = -35$$

$$+6A = -35 + 11 = -24, \therefore A = 4$$

$$\therefore 4+B+C=2 \Rightarrow B+C=2-4 \Rightarrow B+C=-2$$

$$4+4B-C=-9 \Rightarrow 4B-C=-9-4 \Rightarrow 4B-C=-13$$

$$B+C=-2 \dots (5)$$

$$+ 4B-C=-13 \dots (6)$$

$$\frac{15B}{5} = \frac{-153}{5}, \therefore B = -3 \text{ (put in equation 5)}$$

Finally,

$$-3 + C = -2$$

$$C = -2 + 3$$

$$C = 1$$

$$\therefore A=4, B=-3, C=1 \therefore A=4, B=-3, C=1$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \int \frac{4}{x+1} dx + \int \frac{-3}{x-2} dx + \int \frac{1}{x+3} dx$$

$$\begin{array}{lll} \text{let } u = x+1 & u = x-2 & u = x+3 \\ du = dx & du = dx & du = dx \end{array}$$

$$\Rightarrow 4 \int \frac{du}{u} - 3 \int \frac{du}{u} + \int \frac{du}{u}$$

$$\Rightarrow 4 \ln u - 3 \ln u + \ln u$$

$$\Rightarrow 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)$$