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MAT 104 ASSIGNMENT

1 $\int \frac{11-3x}{x^2+2x-3}\,dx$

$x^2+2x-3 = (x+3)(x-1)$

$\frac{11-3x}{x^2+2x-3} = \frac{A}{x+3} + \frac{B}{x-1}$

$\frac{11-3x}{x^2+2x-3} = \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}$

$11-3x = A(x-1)+B(x+3)$

$F(-3) \Rightarrow 11-3(-3) = A(-3-1)+B(-3+3)$

$\frac{20}{-4} = \frac{-4A}{-4}$

$A = -5$

$F(1) \Rightarrow 11-3(1) = A(1-1)+B(1+3)$

$\frac{8}{4} = \frac{4B}{4}$

$B = 2$

$\int \frac{11-3x}{x^2+2x-3}\,dx = \int \frac{-5}{x+3}\,dx + \int \frac{2}{x-1}\,dx$

$\int \frac{-5}{x+3}\,dx \Rightarrow$ Let $u = x+3$

$du/dx = 1, \; dx = du$

$= \int \frac{-5}{u} du \Rightarrow -5 \int \frac{1}{u} \cdot du$

$= -5 \ln u$

$= -5 \ln(x+3)$

$\int \frac{2}{x-1} dx \Rightarrow$ Let $u = x-1$

$\frac{du}{dx} = 1$, $du = dx$

$\int \frac{2}{x-1} dx = \int \frac{2}{u} du \Rightarrow 2 \int \frac{1}{u} \cdot du$

$= 2 \ln u = 2 \ln(x-1)$

$\therefore \int \frac{11-3x}{x^2+2x-3} = 2 \ln(x-1) - 5 \ln(x+3) + C$

2. $\int \frac{4x-16}{x^2-2x-3} dx$

$x^2-2x-3 = (x-3)(x+1)$

$\frac{4x-16}{x^2-2x-3} = \frac{A}{x-3} + \frac{B}{x+1}$

$\frac{4x-16}{x^2-2x-3} = \frac{A(x+1)+B(x-3)}{(x-3)(x+1)}$

$4x-16 = A(x+1) + B(x-3)$

$F(3) \Rightarrow 4(3)-16 = A(3+1) + B(3-3)$

$12-16 = 4A$

$\frac{-4}{4} = \frac{4A}{4}$

$A = -1$

$F(-1) \Rightarrow 4(-1)-16 = A(-1+1) + B(-1-3)$

$-4-16 = -4B$

$\frac{-20}{-4} = \frac{-4B}{-4}$

$B = 5$

$$\int \frac{4x-16}{x^2-2x-3}\,dx = \int \frac{-1}{x-3}\,dx + \int \frac{5}{x+1}\,dx$$

$\int \frac{-1}{x-3}\,dx \Longrightarrow$ Let $u = x-3$

$dy/dx = 1, \quad du = dx$

$$\int \frac{-1}{x-3}\,dx = \int \frac{-1}{u}\cdot du = -1 \ln u$$
$$= -\ln(x-3)$$

$\int \frac{5}{x+1} \Longrightarrow$ Let $u = x+1$

$dy/dx = 1, \quad du = dx$

$$\int \frac{5}{x+1}\,dx = \int \frac{5}{u}\,du \Longrightarrow 5\int \frac{1}{u}\cdot du$$
$$= 5 \ln u$$
$$= 5 \ln(x+1)$$

$$\therefore \int \frac{4x-16}{x^2-2x-3}\,dx = -\ln(x-3) + 5\ln(x+1) + C$$

or

$$\int \frac{4x-16}{x^2-2x-3}\,dx = 5\ln(x+1) - \ln(x-3) + C$$

3 $\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx$

$$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$2x^2-9x-35 = A(x^2+x-6) + B(x^2+4x+3) + C(x^2-x-2)$$

$$2x^2-9x-35 = Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$2x^2-9x-35 = (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C)$$

$$A+B+C = 2 \quad \text{---} \quad (1)$$

$$A+4B-C = -9 \quad \text{---} \quad (2)$$

$$(-6A+3B-2C) = -35 \quad \text{---} \quad (3)$$

From eqn (1) $\Rightarrow A = 2-B-C \quad \text{---} \quad (4)$

From eqn (2); $2-B-C+4B-C = -9$

$$4B-B-C-C = -9-2$$

$$3B-2C = -11 \quad \text{---} \quad (5)$$

From eqn (3), $-6(2-B-C) + 3B - 2C = -35$

$$-12+6B+6C+3B-2C = -35$$

$$6B+3B+6C-2C = -35+12$$

$$9B+4C = -23 \quad \text{---} \quad (6)$$

Using Elimination method btw eqn ⑤ and eqn ⑥

$$3B - 2C = -11$$
$$9B + 4C = -23$$

multiplying eqn ⑤ by 9 and Eqn ⑥ by 3, we have

$$27B - 18C = -99$$
$$-27B + 12C = -69$$
$$-30C = +30$$
$$C = 1$$

Substitute $C = 1$ into eqn ⑤

$$3B - 2(1) = -11$$
$$3B - 2 = -11$$
$$3B = -11 + 2$$
$$3B = -9$$
$$B = -3$$

Finally substitute $B = -3$, and $C = 1$ into eqn ①

$$A + B + C = 2$$
$$A + (-3) + 1 = 2$$
$$A - 3 + 1 = 2$$
$$A = 2 + 3 - 1$$
$$A = 4$$

$$\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx = \int \frac{4}{x+1}\,dx + \int \frac{-3}{x-2}\,dx + \int \frac{1}{x+3}\,dx$$

$\int \frac{4}{x+1}\,dx \Rightarrow$ Let $u = x+1$,

$du/dx = 1$, $du = dx$

$\int \frac{4}{x+1} = \int \frac{4}{u}\cdot du = 4\ln u = 4\ln(x+1)$

$\therefore \int \frac{-3}{x-2}\,dx = -3\ln u = 3\ln(x-2)$

$\int \frac{1}{x+3}\,dx = \ln(u) = \ln(x+3)$

$\therefore \int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + C$